Incentives and Game Theory

Jeffrey Ely

April 15, 2010

Putting Utilitarianism to Work

Example

Suppose that you and your roomate are considering buying an espresso machine for your apartment. The machine costs \$50. Your willingness to pay is $v_1 = 40$. You know that your roomate has a willingness to pay v_2 but you don't know what it is. (Your values are zero when you do not purchase the machine.)

You and your roomate are choosing among the following alternatives.

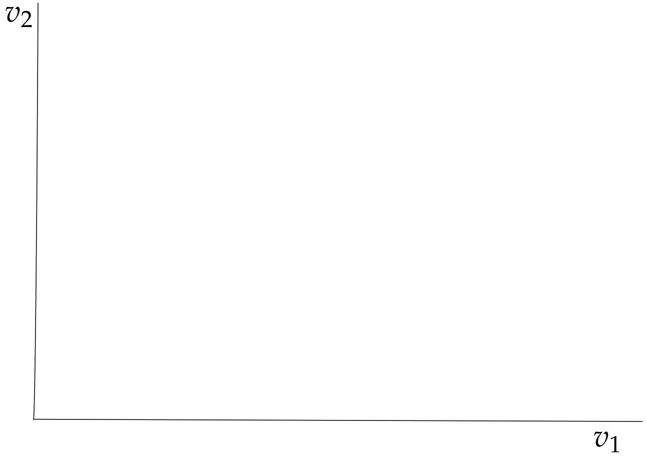
1. no machine, no monetary payments.
2. espressso machine, any transfer scheme $t = t_1, t_2$ such that $t_1 + t_2 = 50$.

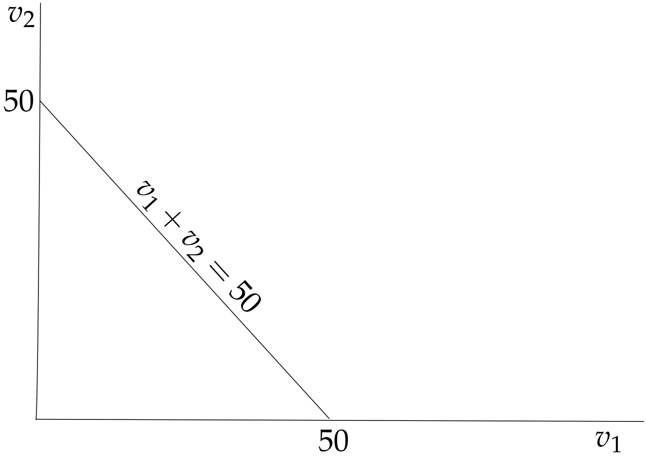
Utilitarianism in Your Apartment

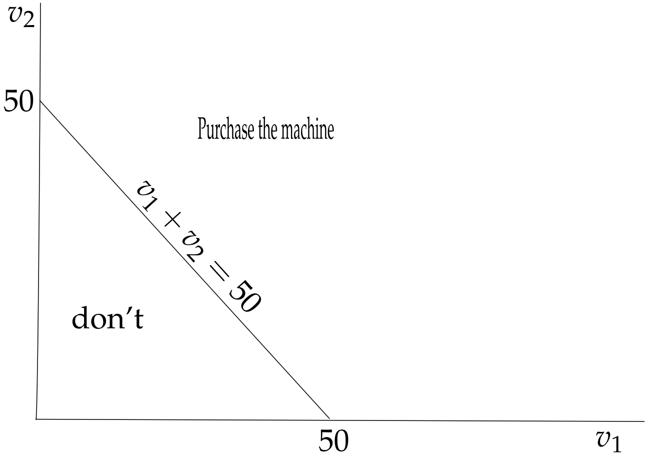
The utilitiarian policy is to purchase the espresso machine if

$$v_1 + v_2 \geq 50$$

and not to purchase the machine if $v_1 + v_2 < 50$.


v_2
v_1

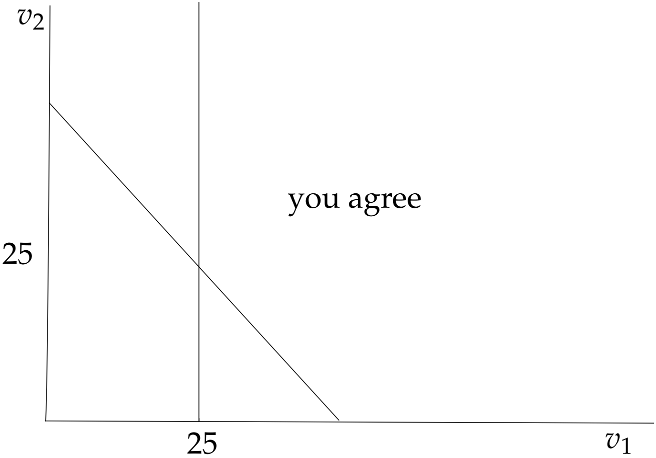

v_2
50
$v_1 + v_2 = 50$
50
v_1


v_2
50
Purchase the machine
$v_1 + v_2 = 50$
don't
50
v_1

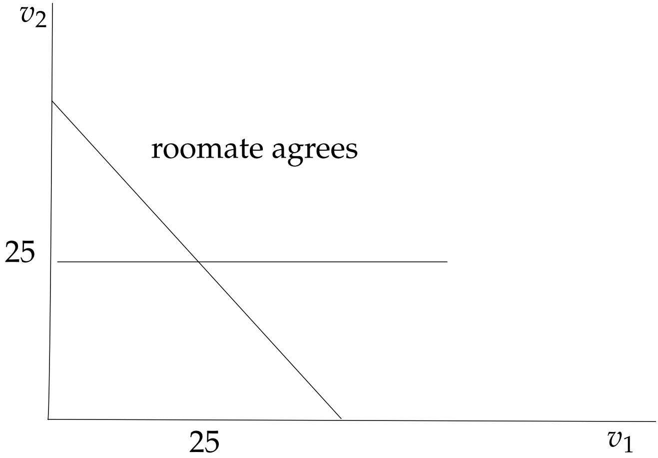
Split the Cost?

- Suppose you agree to split the cost.
- When would you be willing to do it?
- Only when $v_1 \geq 25$.
- And the same is true of your roomate.

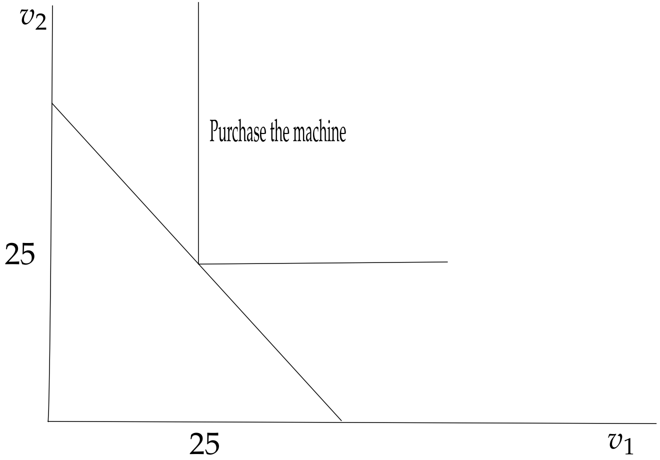
Split the Cost is Inefficient



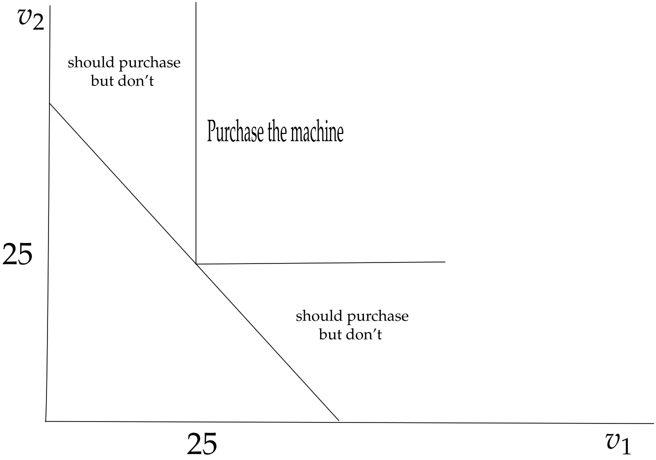
Split the Cost is Inefficient



Split the Cost is Inefficient



Split the Cost is Inefficient



Mechanism Design

Can we devise a mechanism which satisfies the following two conditions

1. gets the two to tell truths about values
2. allows to microwave to be boutght whenever it should

Contribution Game

- The roomates simultaneously pledge a contribution (some number.)
- If the contributions add up to at least 50 then the espresso machine is bought (and the surplus divided proportionally)
- Otherwise not.

Game Theory

A game is described by

1. The players $i = 1, \ldots, n$,
2. The choices available to each of them: A_i. (called the *actions* or *strategies*.)
   - Player i chooses one a_i from A_i.
   - The players choose simultaneously.
3. The *outcomes*: $a = (a_1, a_2, \ldots, a_n)$ where $a_i \in A_i$ for each i.
   - When we write a_{-i}, we mean $(a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots a_n)$.
   - Then (a_i, a_{-i}) is another way of writing a.
4. Utilities: $\pi_i(a)$ (also called *payoffs*.)
   - $\pi_i(a)$ is a number representing the preference for outcome a.
   - Player i wants to choose his action a_i to maximize $\pi(a_i, a_{-i})$.
   - But player i has no control over a_{-i}.

Golden Balls

Yes indeed. Golden Balls

Payoff Matrix

	split	steal
split	50, 50	0, 100
steal	100, 0	0, 0

Dominated Strategies

Definition

Strategy a_i dominates strategy a_i' if a_i always gives a payoff at least as high as a_i' and sometimes strictly higher. In formal terms, a_i dominates a_i' if

1. for every action profile a_{-i} of the opponents,

$$\pi_i(a_i, a_{-i}) \geq \pi_i(a_i', a_{-i})$$

2. and for at least one action profile $\hat{a}_{-i}$ of the opponents,

$$\pi_i(a_i, \hat{a}_{-i}) > \pi_i(a_i', \hat{a}_{-i})$$

Dominant Strategies

Definition

A strategy a_i is *dominant* if it dominates all other strategies.

Super Golden Balls

- Each player has 100 pairs of balls.
- The game goes for up to 100 rounds.
- Each round an additional $100,000 at stake.
- Each time they both say `split`, they each earn $50,000.
- The first round in which either of them say `steal`,
  - The game ends.
  - If only one person said `steal`, that person gets the whole $100,000 from that round.
- If neither chooses `steal` for 100 rounds the game ends. (And by then they have won $5,000,000.)

Iterative Removal of Dominated Strategies

- We begin by removing from consideration the dominated strategies.
- We consider the reduced game that remains.
- Now we remove strategies that are dominated in the reduced game.
- We continue with this until there is nothing more to remove.

Generous Balls

	split	steal
split	50, 50	0, 100
steal	0, 0	1, 0

Not Every Game Has Dominated Strategies

	red	blue
red	$-10, -10$	$1, 0$
blue	$0, 1$	$-10, -10$

Figure: The Dress Dibs Game

What about the contribution game from earlier?