Auctions

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Recap of Dominated Strategies

- Recall that a strategy is dominated if there is another strategy that always does at least as well, and in some cases strictly better.
- A *dominant* strategy is one which dominates all others.
- It is clear that a player should never play a dominated strategy and should play a dominant strategy when it exists.
- However, it is rare that a game has a dominant strategy.
- And removing dominated strategies doesn't always lead to a clear prediction.

For example, recall the negotiation over the cost of the espresso machine.

Mechanism Design

The next part of the course will be about designing games so that the outcome is the utilitarian solution. Recall that the difficulty is

- Inducing the players to reveal their values,
- And then enacting the utilitarian decision given those values.

We will mostly be looking for games which achieve this goal when the players play dominant strategies.

The Simplest Choice Problem: Allocate a Prize to One Individual

- Who wants to fill Obama's seat in the Senate?
- Who gets to fill Obama's seat in the Senate?
- Let's Hold an Auction!

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First-Price Sealed-Bid Auction

- In a sealed-bid acution, bidders (the players in the game) simultaneously submit bids.
- The bids are final.
- In a first-price auction, the winner is the high bidder and pays his bid.
- Losers do not pay.

An Auction as a Game

- The players are the bidders.
- The actions are the bids. (Here we write b_i instead of a_i.)
- The payoffs:
  - $v_i - p$ if you win, where p is the winning price.
  - 0 if you lose.

The Blagojevich Auction

Essentially a first-price sealed bid auction.

Dominated Strategies

- Suppose your value is $v_i > 0$, what should you bid?
- You should not bid v_i or anything higher.
  - Bidding v_i is dominated by bidding $v_i/2$.
  - With a bid of v_i you will never have a positive payoff, even if you win.
  - With a bid of $v_i/2$ you will have a positive payoff if you win (and your payoff will never be negative.)
  - Bidding more than v_i is even worse..

No Dominant Strategy

Any bid less than v_i would not be dominated.

- Take a bid b_i such that $0 \leq b_i < v_i$.
- Is there another bid b_i' which dominates b_i?
  - If $b_i' > b_i$ then b_i does better when the opponents all bid less than b_i.
  - If $b_i' < b_i$ then b_i does better when the opponents all bid between b_i' and b_i.
- Since no other bid does at least as good against all bids of the opponent, b_i is not dominated.

The bottom line is that you should bid less than your value in a first-price sealed-bid auction.

- How much less depends on what you think others' will bid.
- Blagojevich evidently could not get anyone to bid more than \$2.5 million.
- The value of a Senate Seat has been estimated at \$6.2 million.

Second-Price Auction

- Suppose that the winner doesn't pay his own bid, but instead the second-highest bid.
- Apart from that the rules are the same as the first-price auction.
- What would you bid?

Bidding Your Value is a Dominant Strategy

In the second-price auction, it is a dominant strategy to bid exactly your true value v_i.

- Bidding $b_i = v_i$ dominates bidding $b_i < v_i$.
- To show this we must show first that b_i does at least as well against any bid profile b_{-i}.
  - Suppose the highest bid among the opponents ($\max b_{-i}$) is larger than v_i.
    - Then both bids lose.
    - and so the payoff is the same (zero).
  - Suppose $\max b_{-i}$ is between v_i and b_i'.
    - Then bidding $b_i = v_i$ wins while bidding b_i' loses.
    - $b_i = v_i$ gives a positive payoff $(v_i - \max b_{-i})$ while b_i' gives zero.
  - Finally, suppose $\max b_{-i}$ is below b_i'.
    - Then both bids win at a price of $\max b_{-i}$.
    - The payoff is the same with either bid $(v_i - \max b_{-i})$.
  - We have shown that $b_i = v_i$ never does worse than $b_i' < v_i$ and in one case, strictly better.

Bidding your Value is a Dominant Strategy

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- To show this we must show that b_i does at least as well against any bid profile b_{-i}.
  - Suppose the highest bid among the opponents $(\max b_{-i})$ is larger than b_i.
    - ★ Then both bids lose.
    - ★ and so the payoff is the same (zero).
  - Suppose $\max b_{-i}$ is between v_i and b_i'.
    - ★ Then bidding $b_i = v_i$ loses while bidding b_i' wins.
    - ★ b_i' gives a negative payoff $(v_i - \max b_{-i})$ while $b_i = v_i$ gives zero.
  - Finally, suppose $\max b_{-i}$ is below v_i.
    - ★ Then both bids win at a price of $\max b_{-i}$.
    - ★ The payoff is the same with either bid $(v_i - \max b_{-i})$.
  - We have shown that $b_i = v_i$ never does worse than $b_i' > v_i$ and in one case, strictly better.

The Vickery Auction

This second-price auction format, and this logic, was discovered by the Nobel-prize winning economist William Vickrey.

The auction is therefore commonly referred to as the Vickrey auction.

What it Means

As a consequence of these observations, when a Vickery auction is used,

- All bidders bid their true value, therefore
- the bidder with the highest value is sure to win, therefore
- the auction allocates the prize efficiently..

In this class, a game which induces the players to truthfully reveal their values and which results in the utilitarian social choice will be called an *efficient mechanism.*

The English Auction

- In the English auction, the price is continuously raised and the last bidder standing is the winner at the final price.
- At what price should you drop out of an English auction if your maximum willingness to pay is v_i?
- Answer: At exactly v_i.
- Thus, the English auction is essentially the same as the Vickrey auction.
- The English auction is widely used in practice.

Extensions of the Vickrey auction idea

- Auctions for multiple items.
- “Reverse” auctions
- Auctions with reserve prices.

However

The Vickrey auction is an efficient mechanism only under certain circumstances.

- No externalities.
- "Private" Values.

We will consider examples to illustrate this.

Externalities

Three bidders x, y, z. The columns of the following table correspond to the three alternatives: allocating the object to either x, y, or z. The rows show each bidder's value for each alternative. So in particular, bidder y has a negative value when z gets the object. This is a *negative externality.*

	X	Y	Z
x	v_x	0	0
y	0	v_y	-5
z	0	0	v_z

In a Vickrey auction, bidder y does not have a dominant strategy. She might even want to bid more than v_y.

Common Values

- In a common values auction, bidders have unique private information about the value of the item up for sale.
- So no individual bidder knows exactly what the object is worth.
- When you win, you infer that others' had information that the value was low.
- This is the *winner's curse.*
- Fear of the winner's curse can lead bidders to be very cautious and may even lead to a breakdown of the market.
- "Toxic assets."