Second-Best Mechanisms

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The VCG mechanism is the Only Efficient Mechanism

- Since the VCG mechanism is the only mechanism that
  - Makes truth-telling a dominant strategy
  - Implements the utilitarian rule
- And since the VCG mechanism yields a budget deficit,
- *There is no budget balanced, efficient mechanism for this social choice problem.*
- Ok then, the “first-best” is not attainable. What’s the best we can do with a budget-balanced mechanism? (The “second-best.”)
Budget Balanced Mechanisms

- From now on, we will consider mechanisms which never run a deficit.
- We will try to find the best among such mechanisms.
- i.e. closest to efficiency.
Two individuals, with willingness to pay $v_1$, $v_2$ for a public good.
The cost to produce the public good is $c$.
It is efficient to produce the good whenever $v_1 + v_2 \geq c$.
But efficiency is not attainable without a budget deficit.
As we look for second-best mechanisms, we now can choose both
- The decision rule (in what cases to produce the good)
- The transfer rule.
Mechanisms

- Alternatives are on (the good is produced) and off (the good is not produced.)
- A mechanism is described by two functions
  - The decision rule, denoted $\alpha(\hat{v})$, which specifies the alternative.
  - The transfer rule, denoted $t(\hat{v})$, which specifies the transfer scheme.
- The only feasible mechanisms are those with $t_1(\hat{v}) + t_2(\hat{v}) \leq 50$ for all $\hat{v}$.
- We also want our mechanisms to be incentive compatible, that is we want truthtelling to be a dominant strategy.
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\[ v_1 + v_2 = c \]

The public good problem.
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A picture of a decision rule. We want to see whether a transfer rule can be designed to make this an incentive-compatible mechanism.
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Suppose that 2 announces \( \hat{v}_2 \).

\( v_1 + v_2 = c \)
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What should $1$ be required to contribute if the good is produced? Note that the contribution cannot depend on $1$’s announcement $\hat{v}_1$. 
We will show incentive compatibility requires that the contribution should be $p^*$. 

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If instead it were higher than $p^*$, say at $p$, then...
1 would have an incentive to understate his value when it is between $p^*$ and $p$. (He would rather not have the good than to pay price $p > v_1$.}
Likewise, the contribution cannot be below $p^*$ . . .
Because then 1 would have an incentive to overstate his value when it is between $p$ and $p^*$. (He would rather have the good and pay only $p < v_1$.)
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We use this same procedure to determine 1’s contribution as a function of \( \hat{v}_2 \).

\[
v_1 + v_2 = c
\]
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And for player 2’s contribution as a function of $\hat{v}_1$. 

$\nu_1 + \nu_2 = c$

$\nu_2$

$\nu_1$

$p^*$
Going back to player 1, what happens when his value is to the right of the cloud?
The set in which the good is produced must extend to the northeast.
Now we will figure out if there is a deficit. Suppose they announce $\hat{v}$ in the region where the good is produced.
So 1 has announced $\hat{v}_1$
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And 2 has announced $\hat{v}_2$. 
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We can figure out what 1 should contribute,
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And what 2 should contribute.
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The point $p^*$ indicates the pair of payments $(p_1^*, p_2^*)$. It is below the diagonal line. That means $p_1^* + p_2^* < c$. There is a budget deficit.
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This happened because the set is not contained in any rectangle whose lower-left corner is on the diagonal line.
On the other hand, if we pick a set which is contained inside such a rectangle, there is guaranteed not to produce a deficit, regardless of what the players announce.
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On the other hand, if we pick a set which is contained inside such a rectangle, there is guaranteed not to produce a deficit, regardless of what the players announce.
So, since we know that the set must be included inside a rectangle, we might as well make it the whole rectangle.
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Such a mechanism is budget balanced (exactly.)
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Any rectangle will do. Different rectangles correspond to different ways of dividing the cost.

\[ v_1 + v_2 = c \]
Summary

- Any incentive compatible mechanism which does not run a deficit must be contained in a rectangle.
- The most efficient of these use up an entire rectangle.
- These are the “second best” mechanisms: the best among those that never run a deficit.
- Any rectangle will is a second-best mechanism.
- The choice of rectangle determines how the cost of the public good will be split.
- Thus, a second-best mechanism reduces to this:
  - *First*, we agree on how the cost would be split.
  - Then the players say yes or no.
  - If one says no, then the good is not produced.
  - If both say yes, then the good is produced and the cost split according to the pre-specified rule.
- This is just the split-the-cost mechanism (for potentially unequal cost shares.)
- We have shown that it is impossible to improve upon that simple mechanism.