Market Mechanisms

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Bilateral Trade

- A single buyer and a single seller
- Seller has one unit of a good.
- Buyer would obtain value $v > 0$ from the good.
- Seller would incur cost $c > 0$ from relinquishing the good.
  - negative value.
  - can also be interpreted as production cost.
- Alternatives are "trade" and "no trade."
- Both have value zero for "no trade"

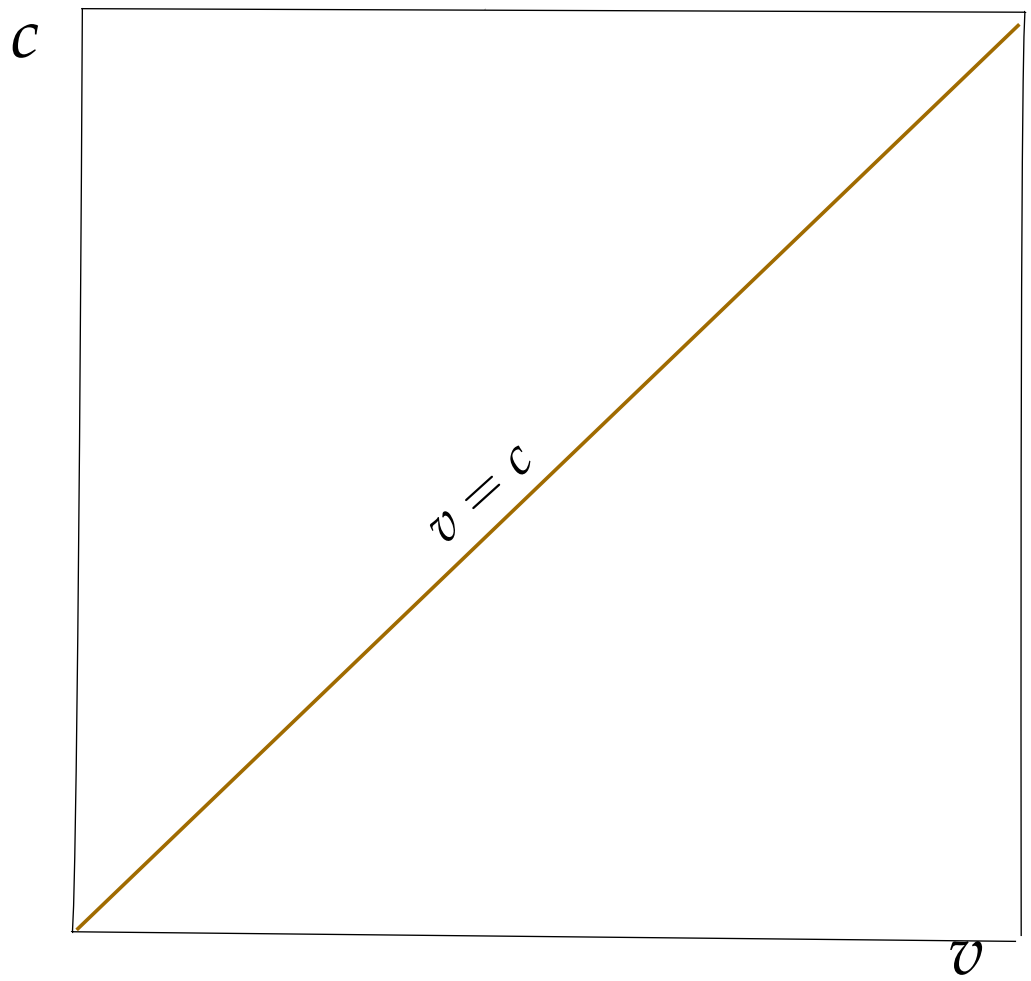
Efficiency

- Trade should occur whenever $v > c$.
- VCG mechanism can implement this rule.
- Transfers in the event of trade:
  - buyer pays c.
  - seller receives v.
  - budget deficit.
  - note that everything is the same if instead the seller has a value v_s from keeping the good and zero value from trade.

Second-best Mechanisms

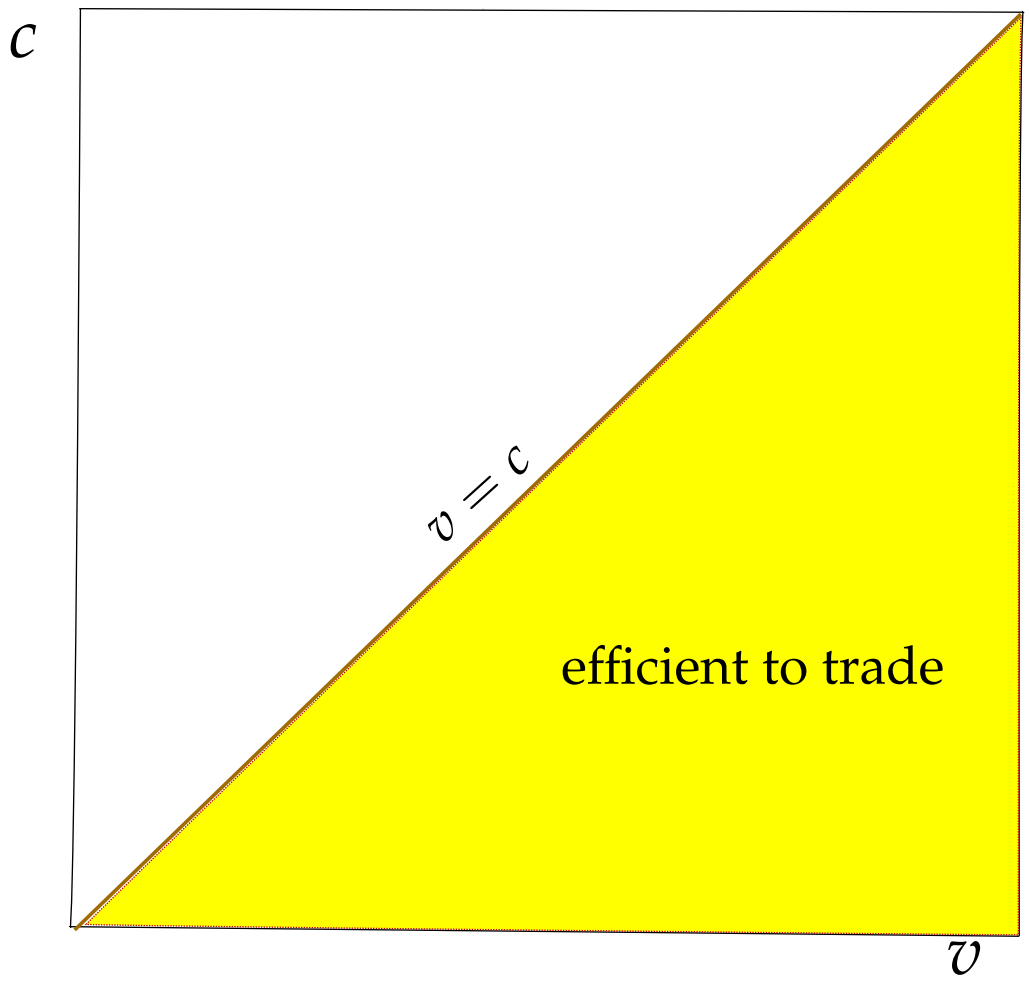
There is a fundamental source of inefficiency in "markets."

Second-best Mechanism for Bilateral Trade



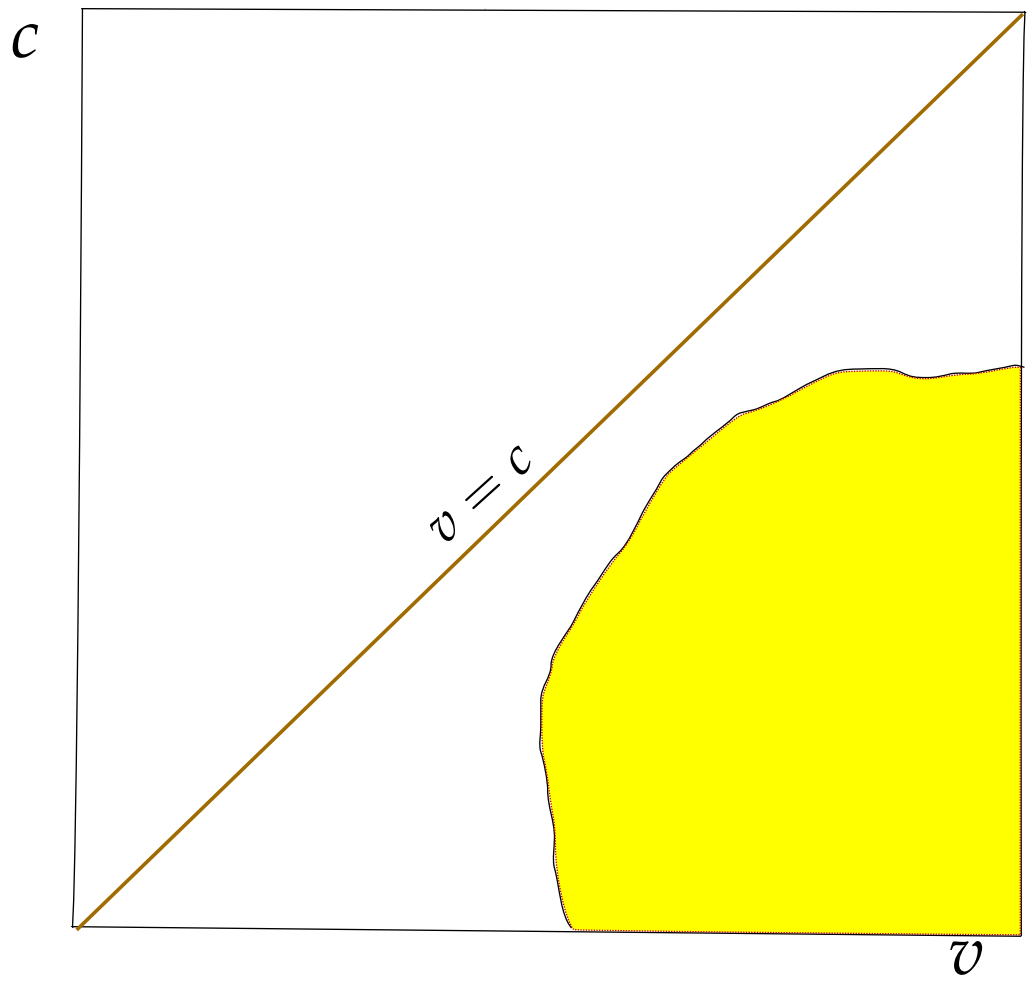
The bilateral trade problem.

Second-best Mechanism for Bilateral Trade



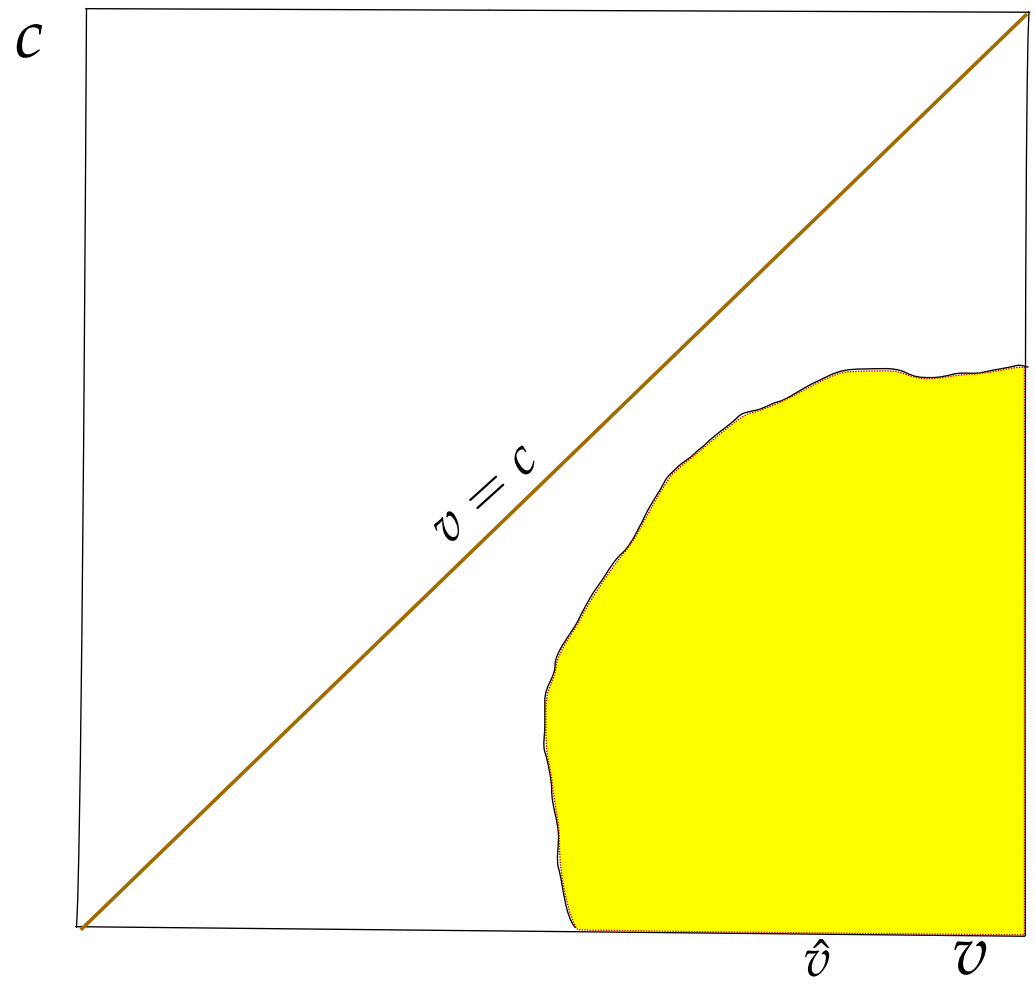
The utilitarian decision rule.

Second-best Mechanism for Bilateral Trade



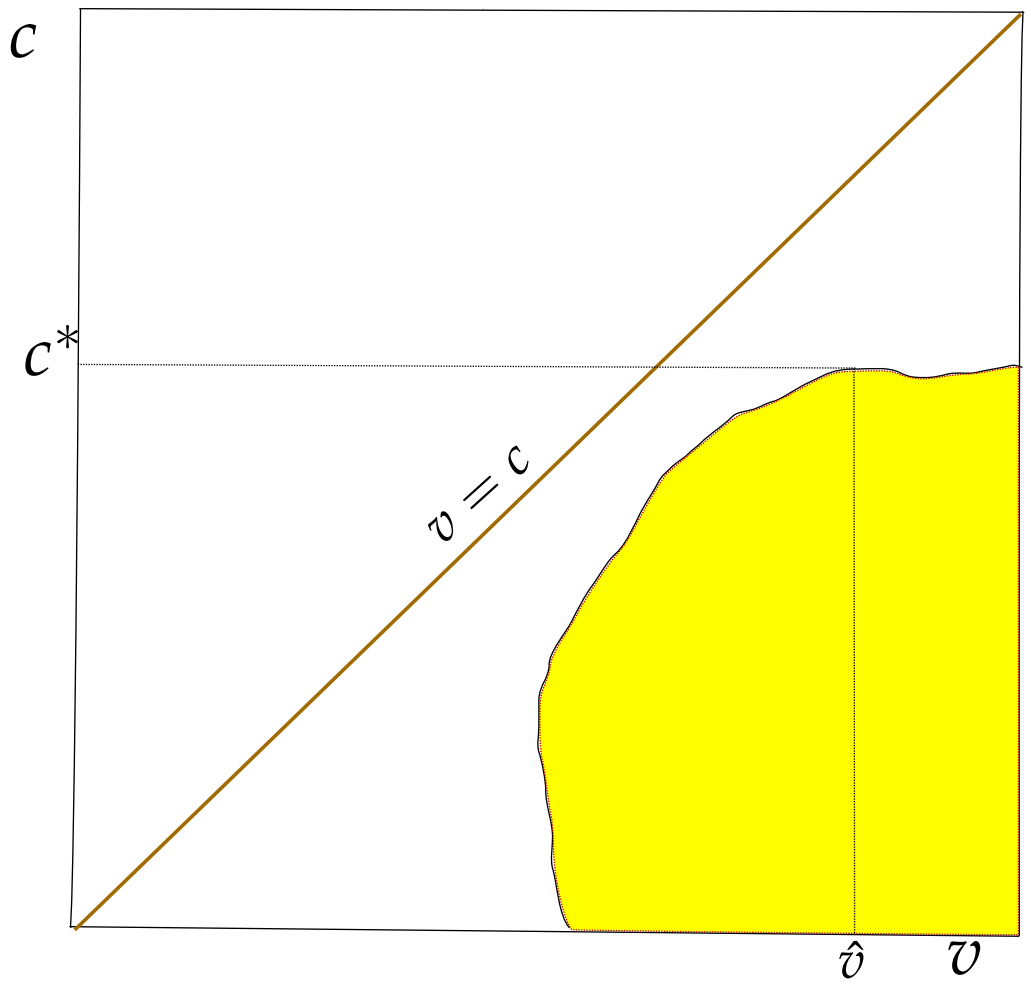
A picture of a decision rule. We want to see whether a transfer rule can be designed to make this an incentive-compatible mechanism.

Second-best Mechanism for Bilateral Trade



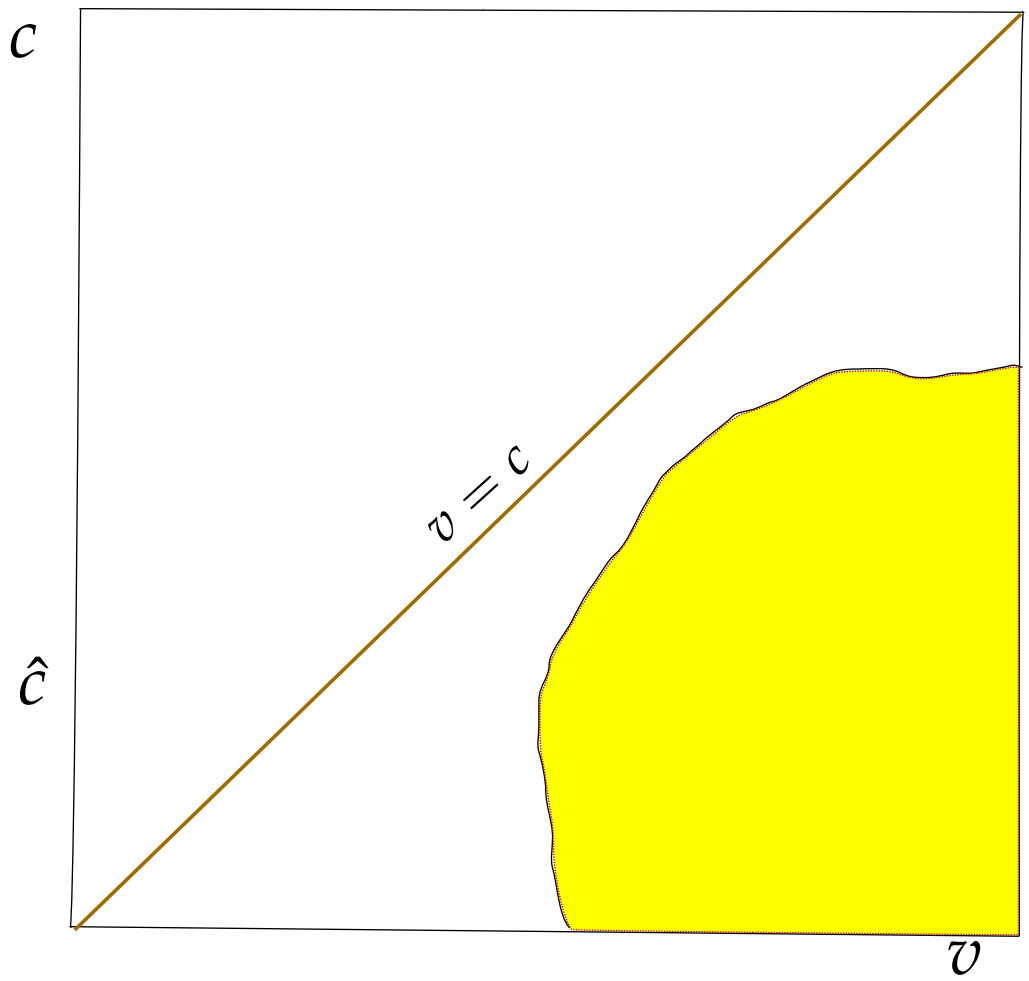
Suppose the buyer announces $\hat{v}$.

Second-best Mechanism for Bilateral Trade



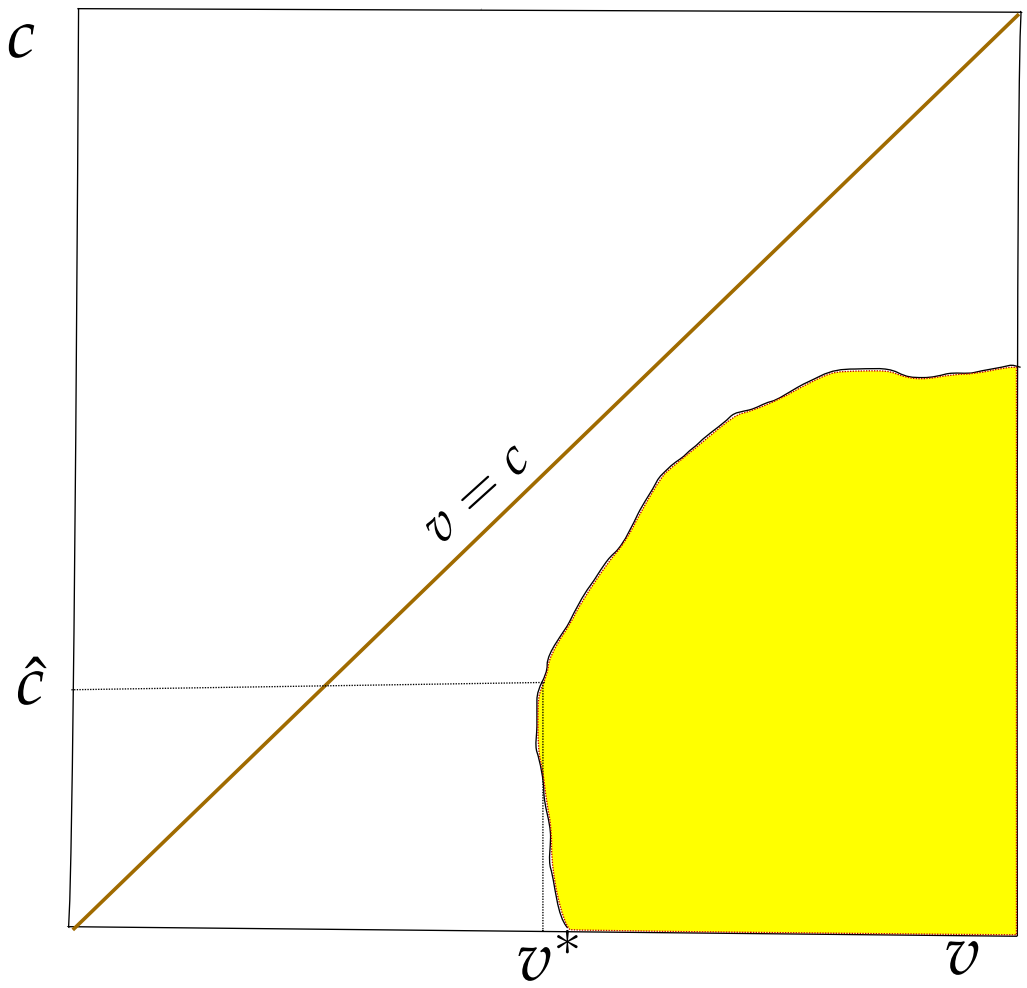
For incentive-compatibility, if there will be trade, the seller must receive c^*.

Second-best Mechanism for Bilateral Trade



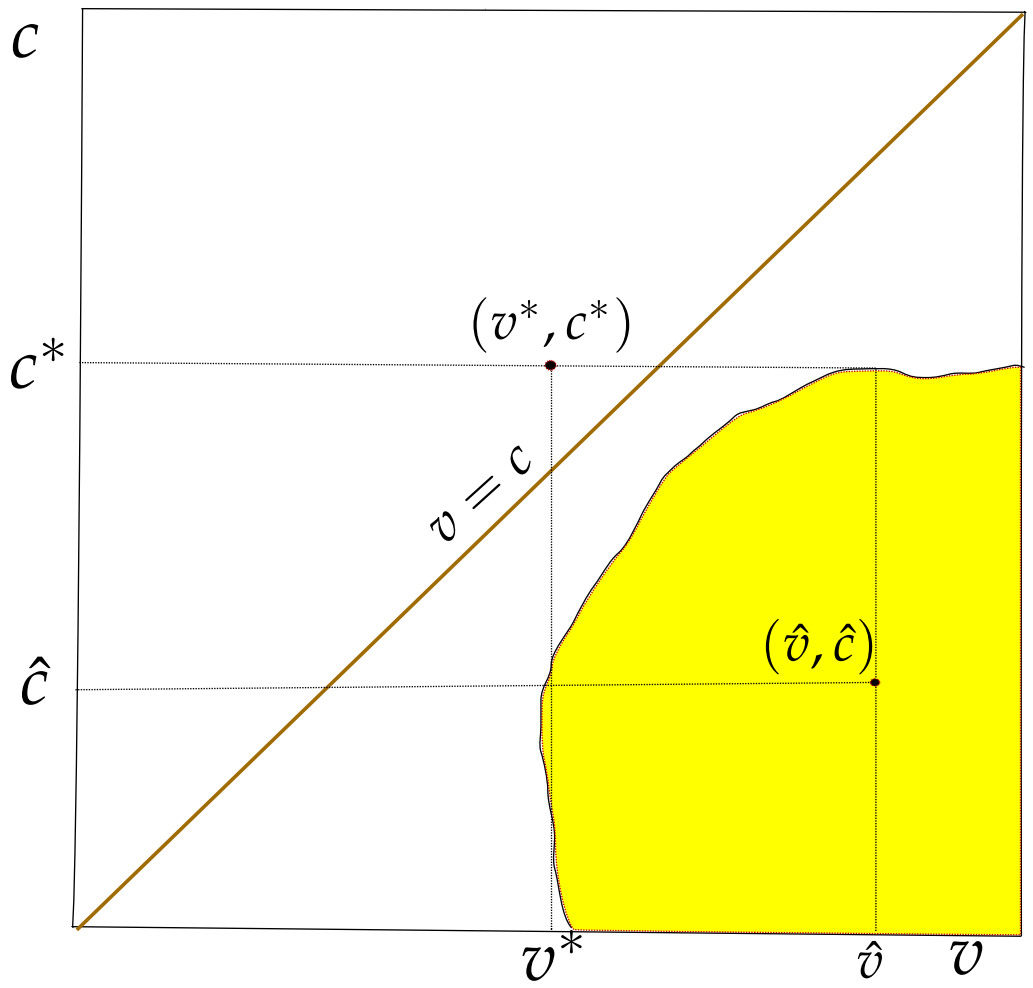
Suppose the seller announces cost $\hat{c}$.

Second-best Mechanism for Bilateral Trade



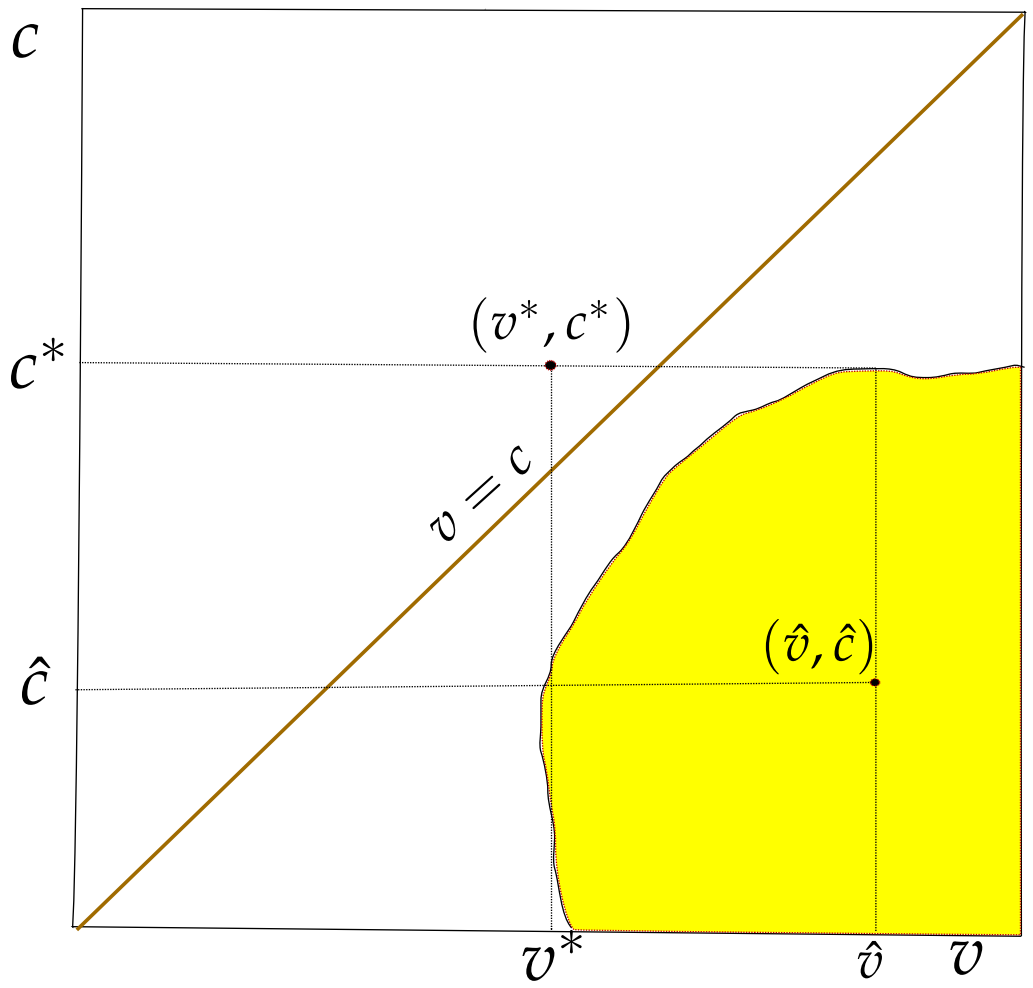
For incentive-compatibility, if there will be trade, the buyer must pay v^*.

Second-best Mechanism for Bilateral Trade



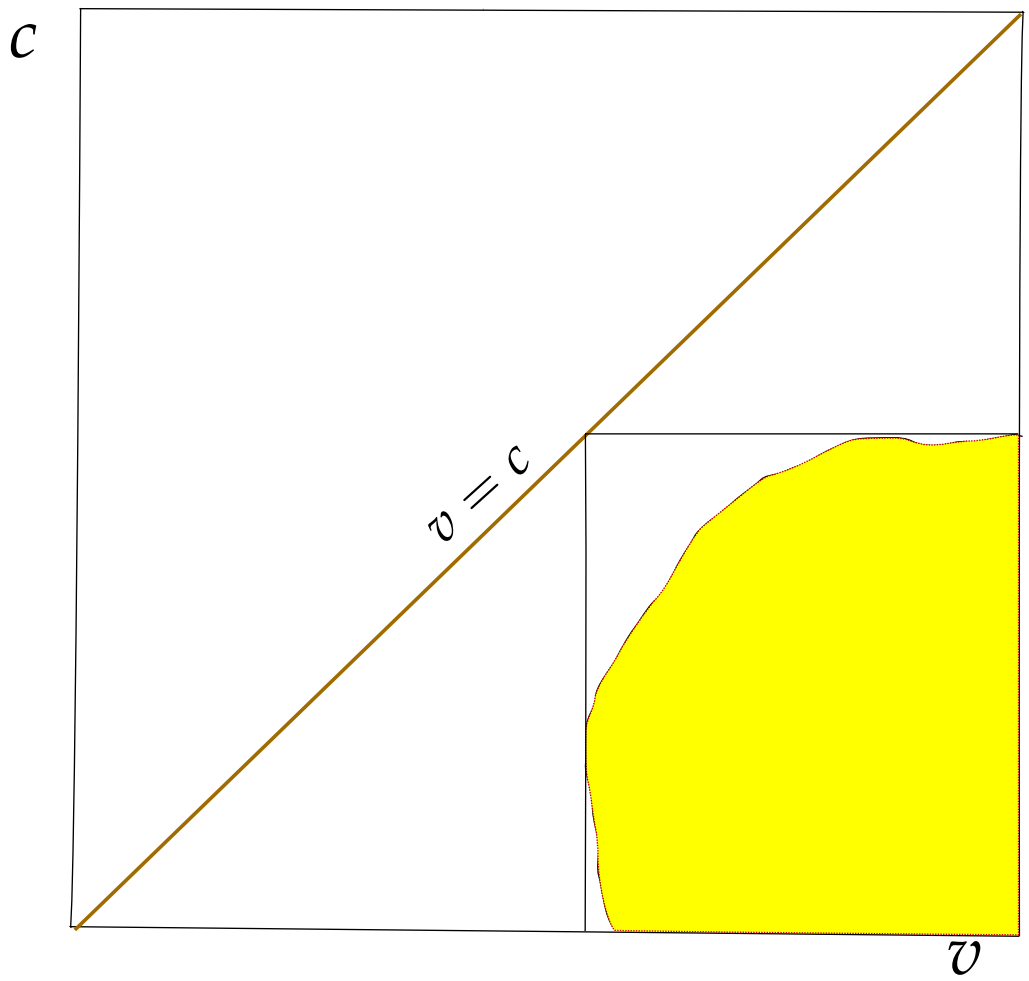
Putting it together, if they announce $(\hat{v}, \hat{c})$, there is a deficit because $c^* > v^*$.

Second-best Mechanism for Bilateral Trade



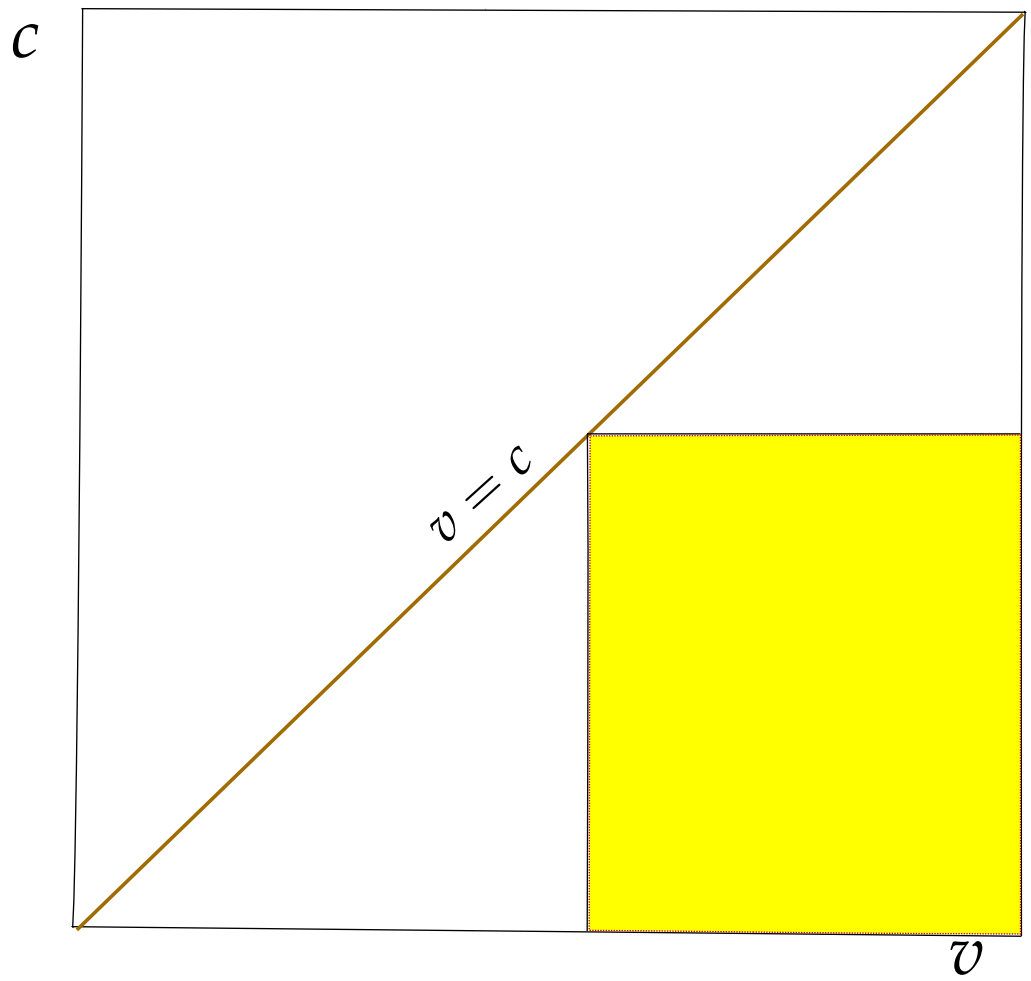
The problem is similar to the problem with public goods.

Second-best Mechanism for Bilateral Trade



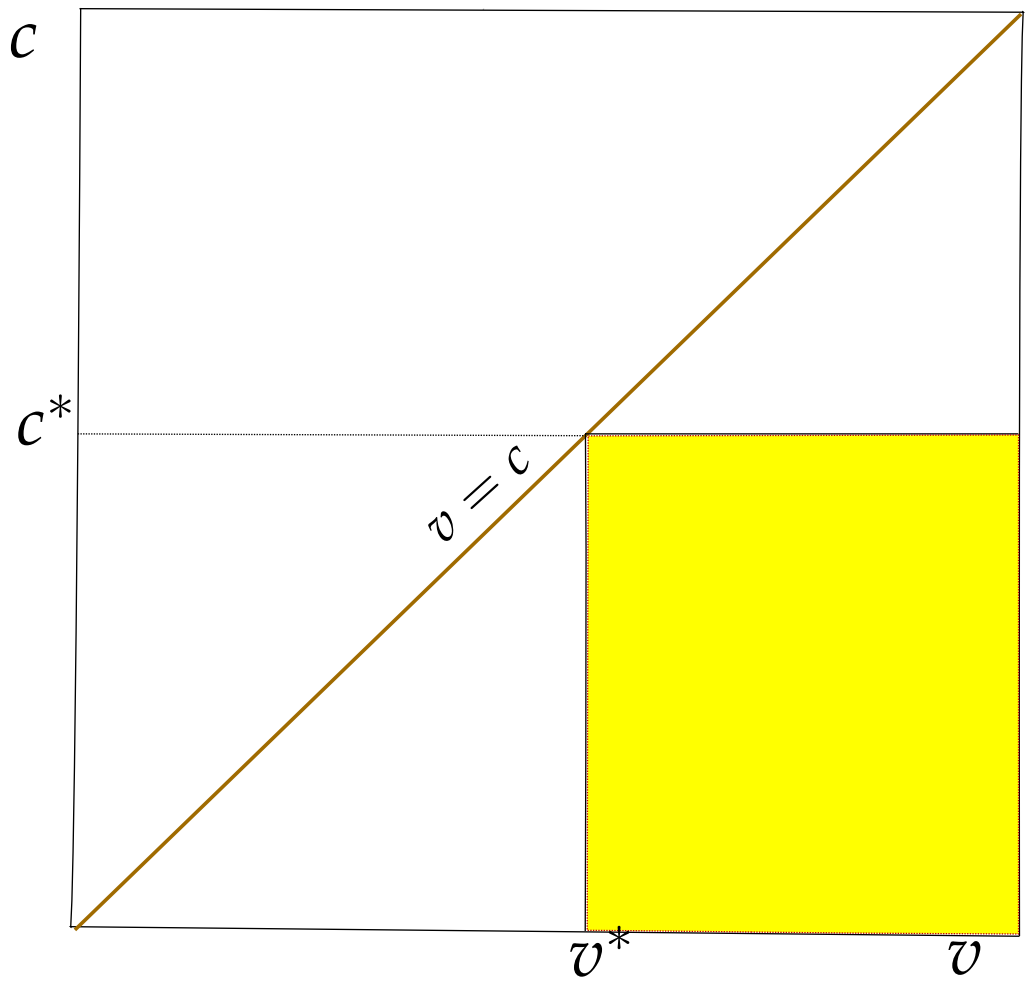
We need the trading set to be included in a rectangle which is on or below the diagonal.

Second-best Mechanism for Bilateral Trade



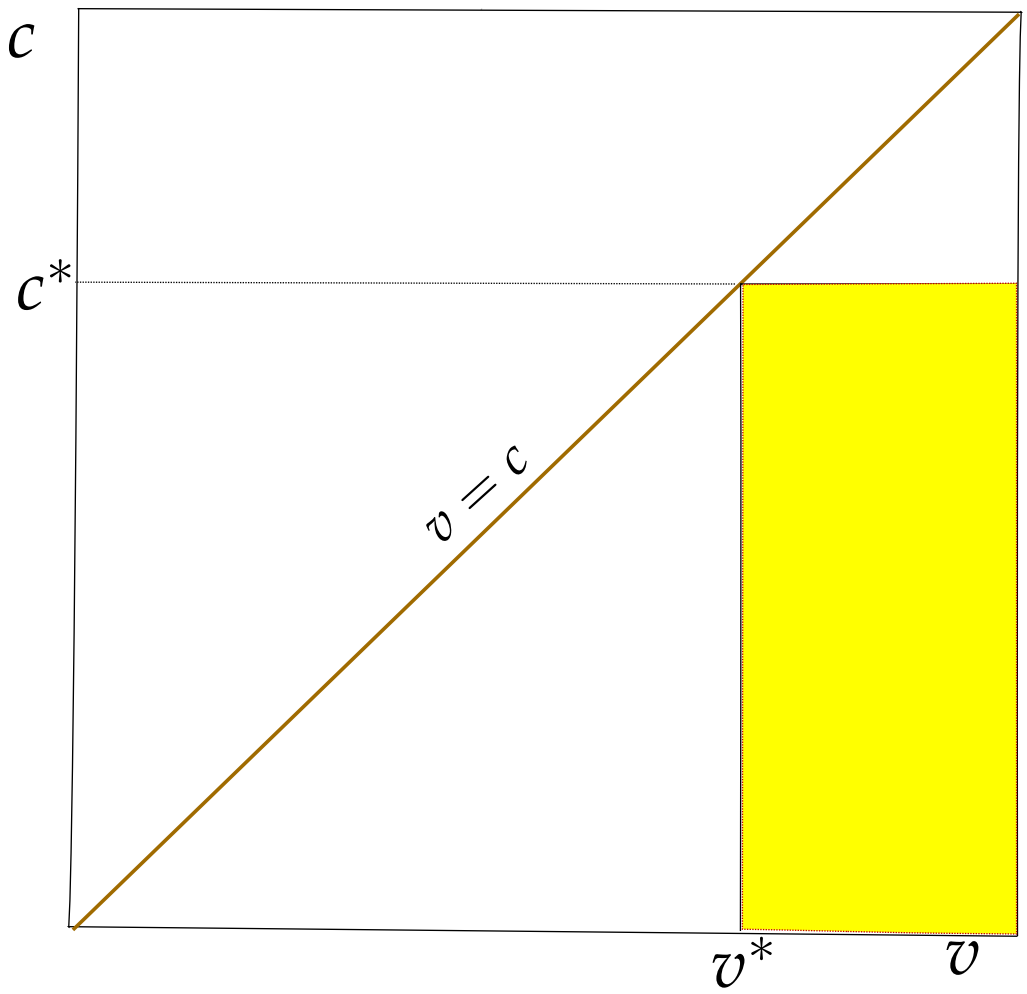
But then we might as well make it the whole rectangle.

Second-best Mechanism for Bilateral Trade



We achieve exact budget balance where the buyer pays v^* and the seller receives $c^* = v^*$. We can call this the *price.*

Second-best Mechanism for Bilateral Trade



Any price will do. But the price is fixed in advance.

Fixed-price Mechanisms

- Any second-best mechanism has a fixed price.
- Thus "markets" are second-best mechanisms.
- However, this theory does not tell us where market prices come from.

Larger Markets

- Suppose now there are two sellers and two buyers.
- Each seller has one unit and his own cost,
- each buyer wants one unit and has his own value.
- How do we organize trade?
- One way is to pair them up.
- But then we still have to pick a price.

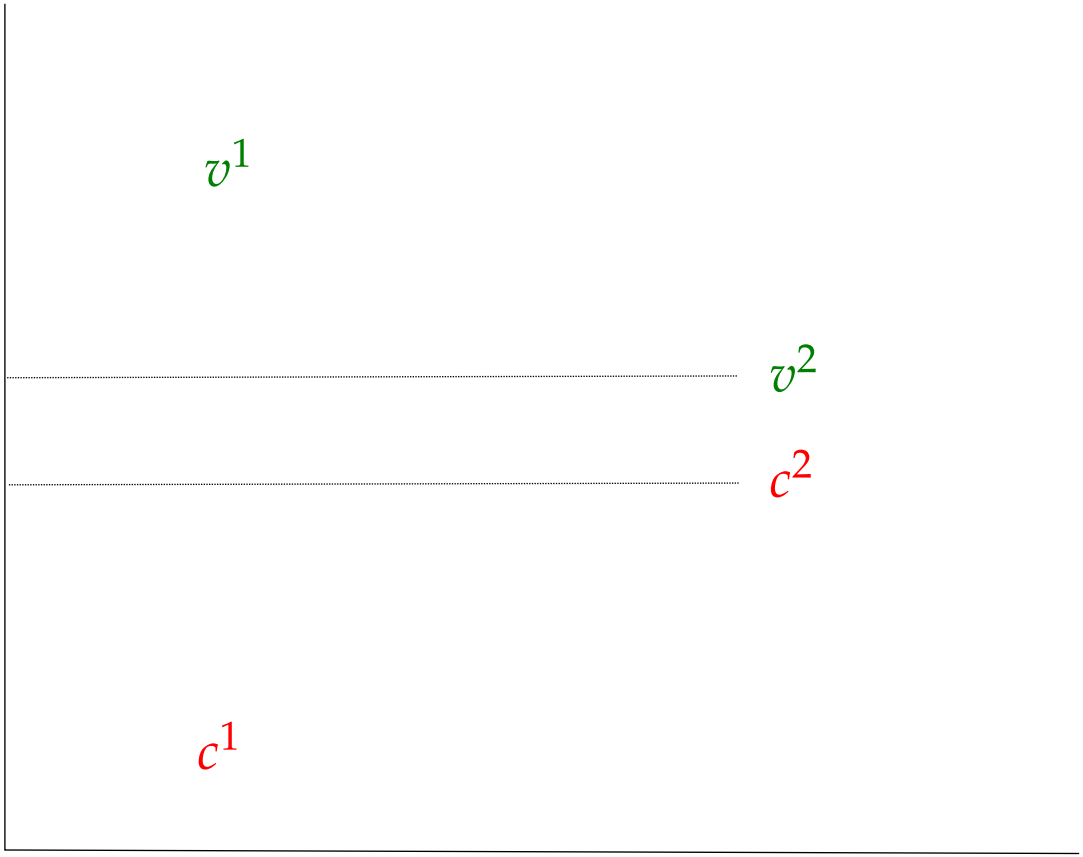
Competition

- We can improve upon separate markets by leveraging *competition.*
- For example, we can hold an auction among the buyers.
  - Buyers announce their values.
  - High bidder will have the right to buy one unit.
  - Price paid is the low bidder's bid.
- And a separate *reverse* auction among the sellers.
  - Sellers announce their costs.
  - Low bidder will have the right to sell one unit.
  - Price received is the higher seller's bid.
- If the buyer's price exceeds the seller's price then a single unit is sold.
- Otherwise, no trade.
- Budget surplus.

Dominant Strategies

- A buyer will not overstate his value.
- A buyer will not understate his value.
- Same for the seller.

Price Discovery



Price Discovery

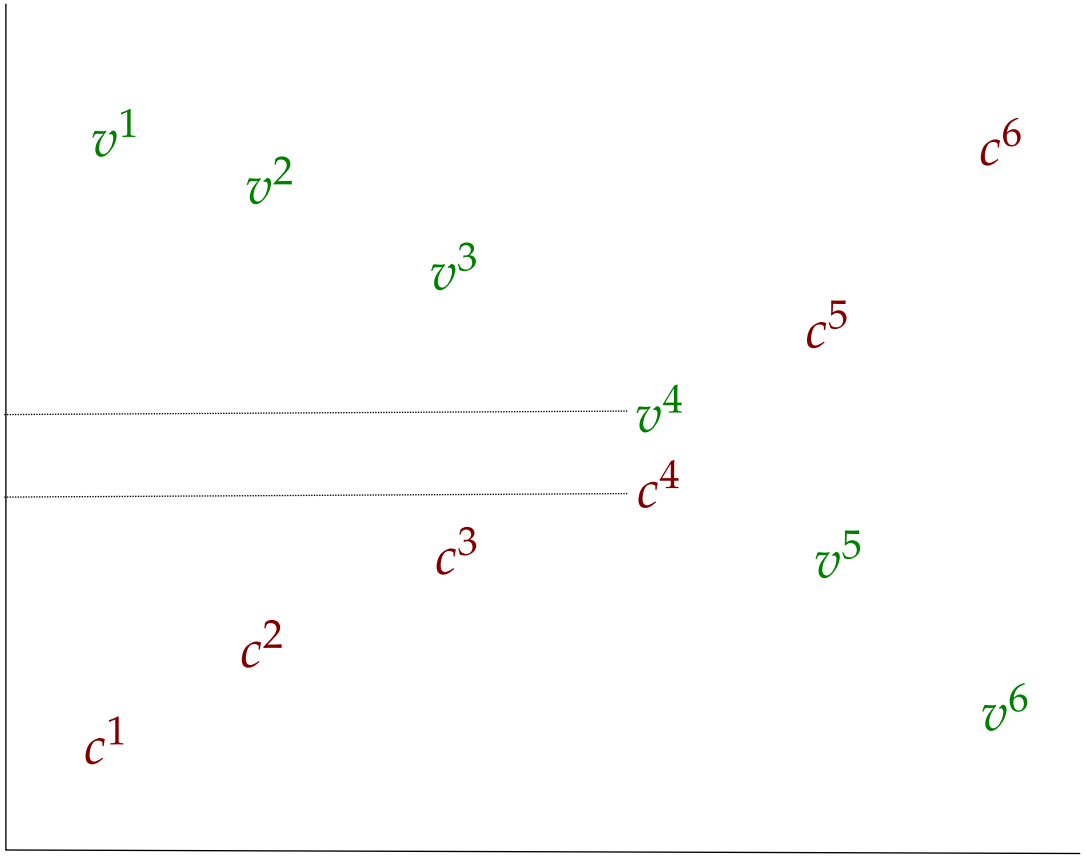
- This mechanism allows the price to be determined by competition.
- If we used a fixed-price mechanism, the price might be to high or too low.

Example: Fixed-price is Too Low

Larger Markets

- All buyers announce their values.
- All sellers announce their values.
- Arrange the buyers in decreasing order of values.
- Arrange the sellers in decreasing order of values.
- Find the number of units that should be traded.

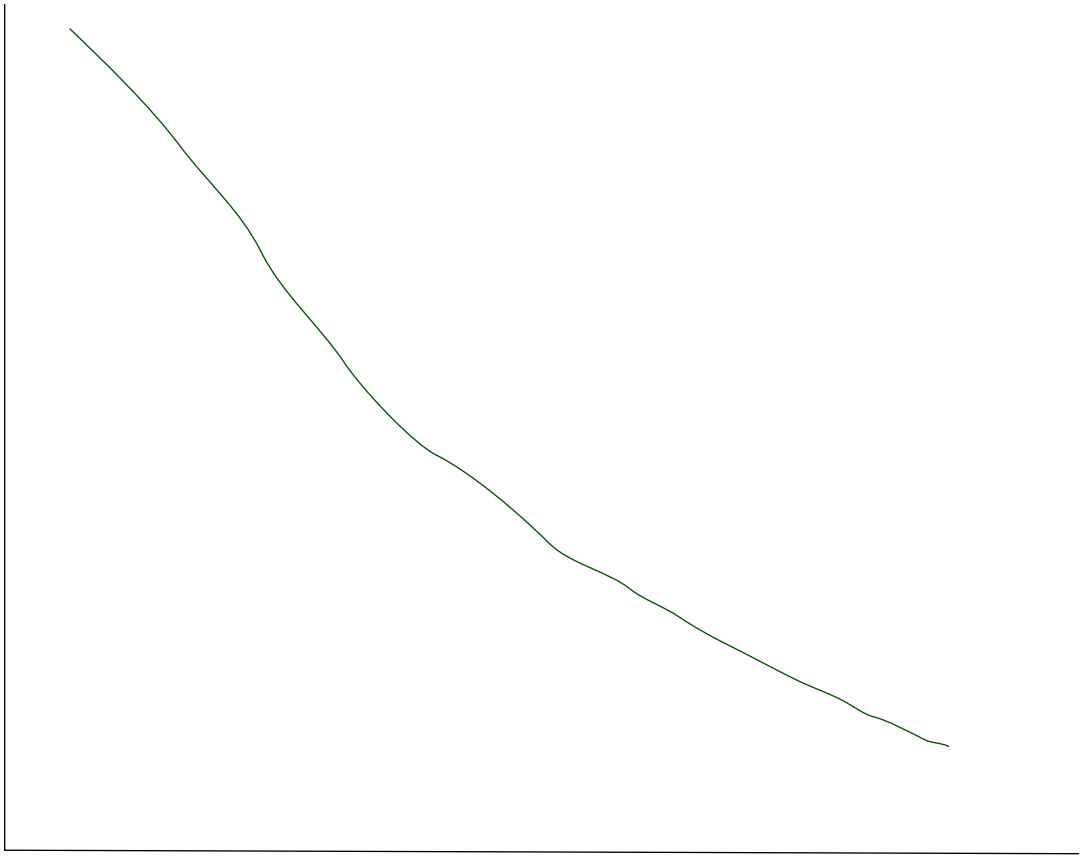
Example: Fixed-price is Too Low



Larger Markets

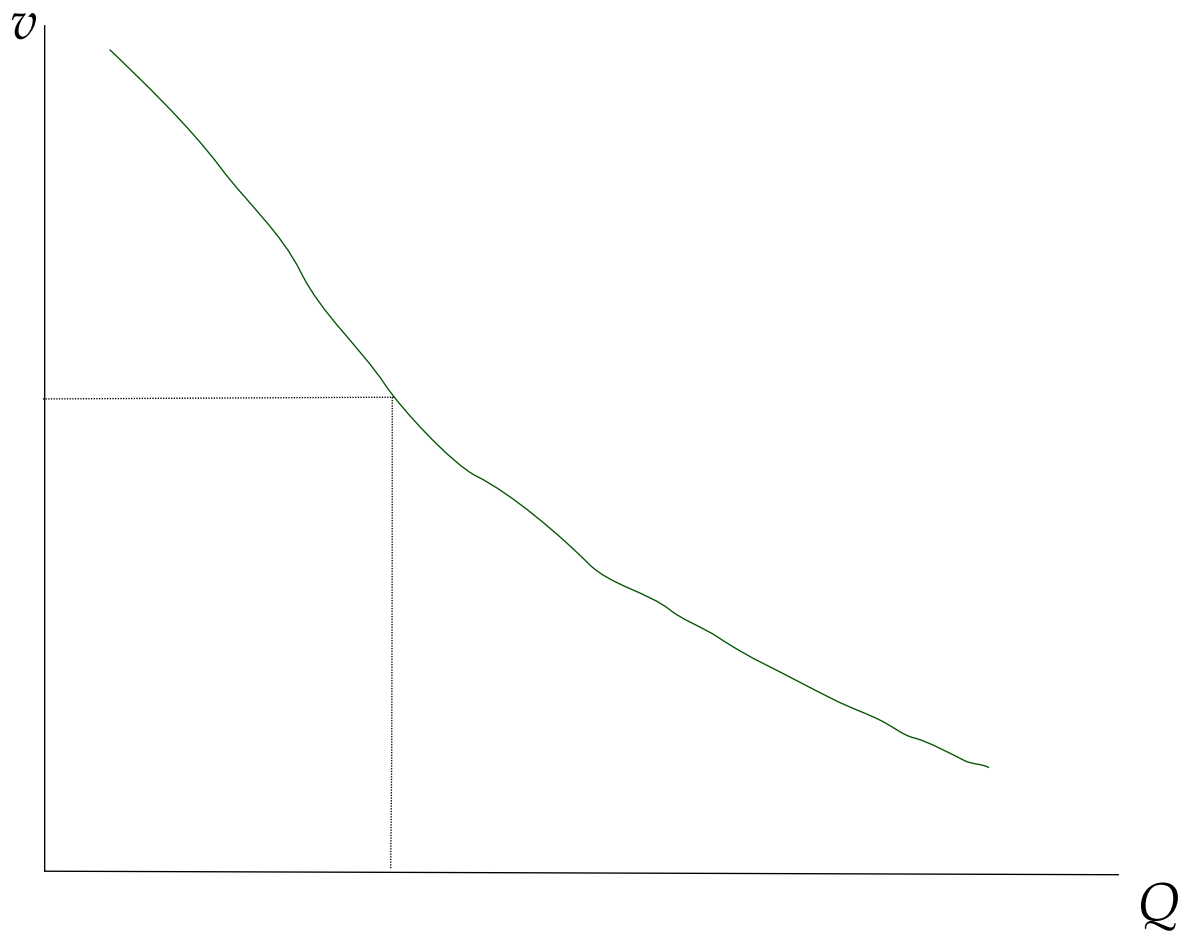
- All buyers announce their values.
- All sellers announce their costs.
- Arrange the buyers in decreasing order of values.
- Arrange the sellers in decreasing order of values.
- Find the number of units k that should be traded.
- Here $k = 4$.
- We will actually trade $k - 1$ units.
- The $k - 1$ buyers with the highest values and the $k - 1$ sellers with the lowest costs will trade.
- All sellers will receive the same price equal to c^k.
- All buyers will pay the same price equal to v^k.
- Since $v^k > c^k$ we have a budget surplus.
- We trade only 1 fewer unit than efficiency demands.

Very Large Markets



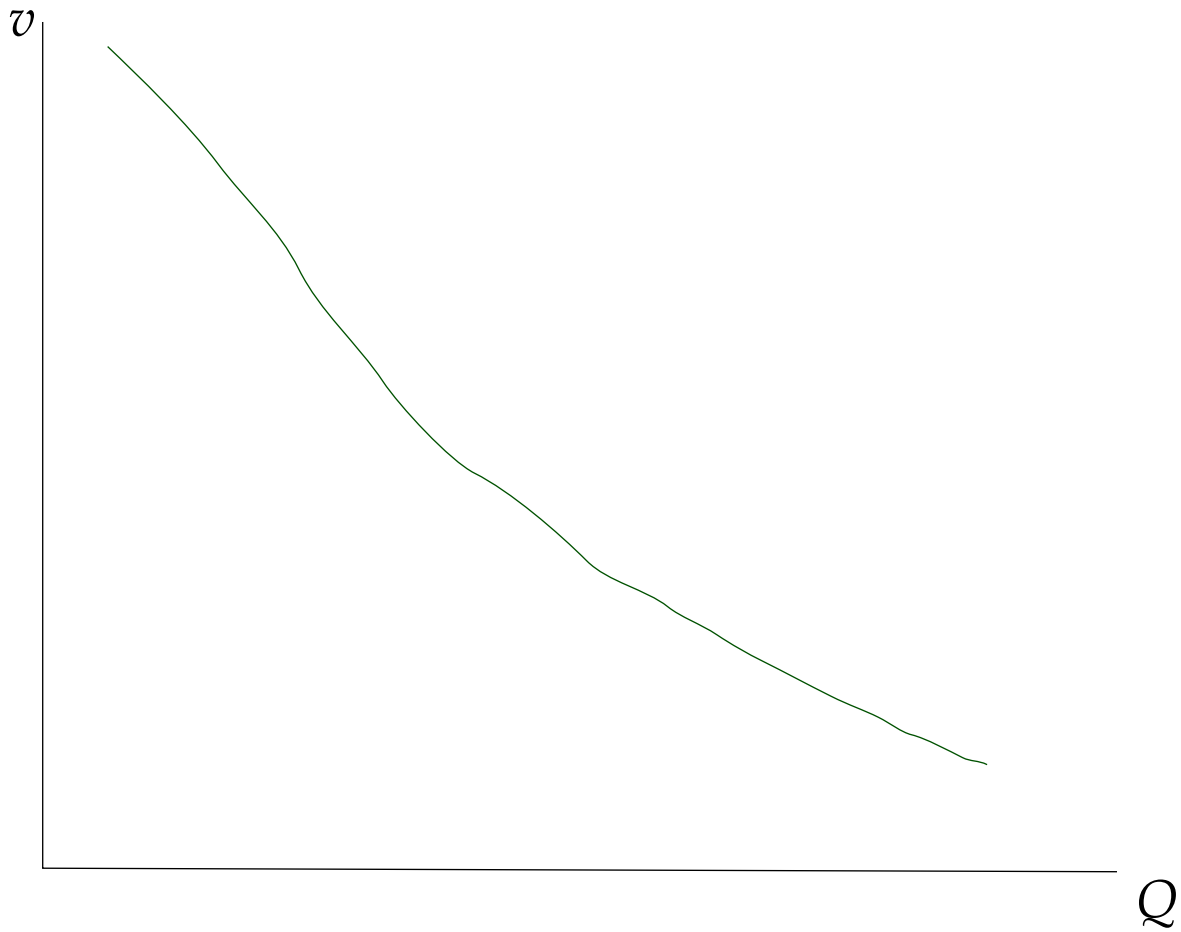
Suppose that there is a large number of buyers.

Very Large Markets



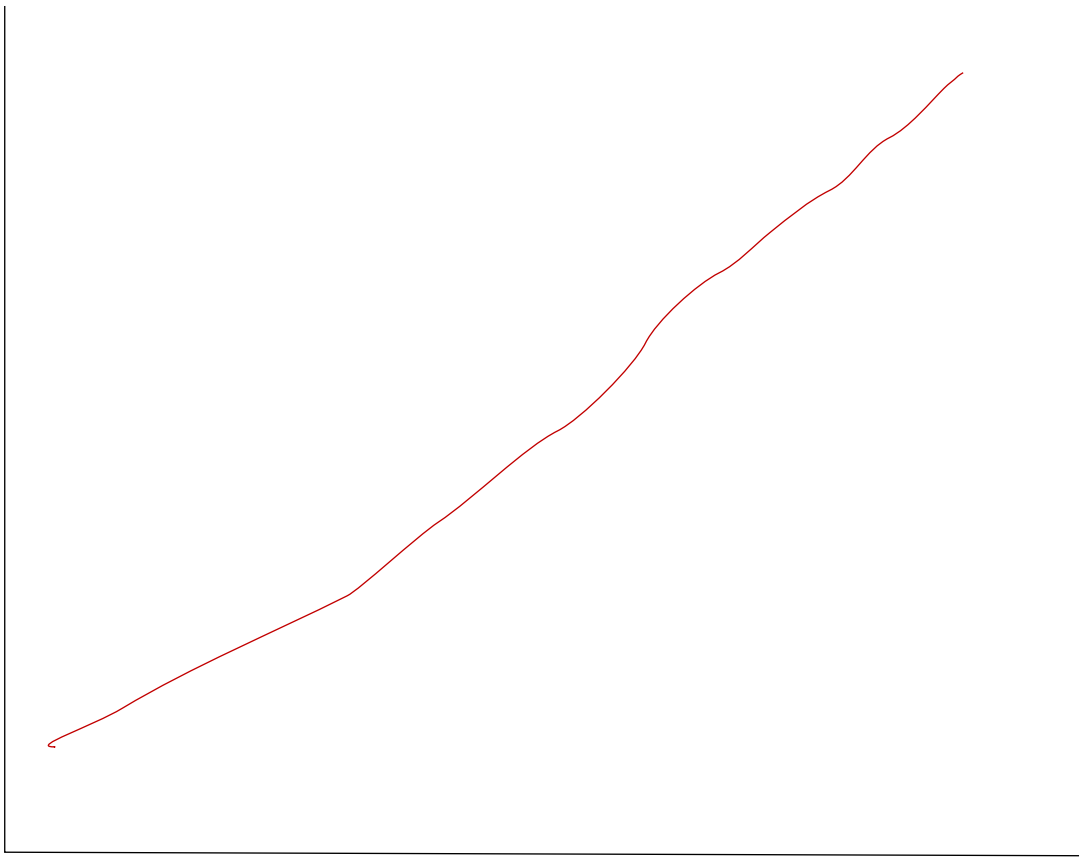
For any value v, we can plot the number of buyers with values greater than v.

Very Large Markets



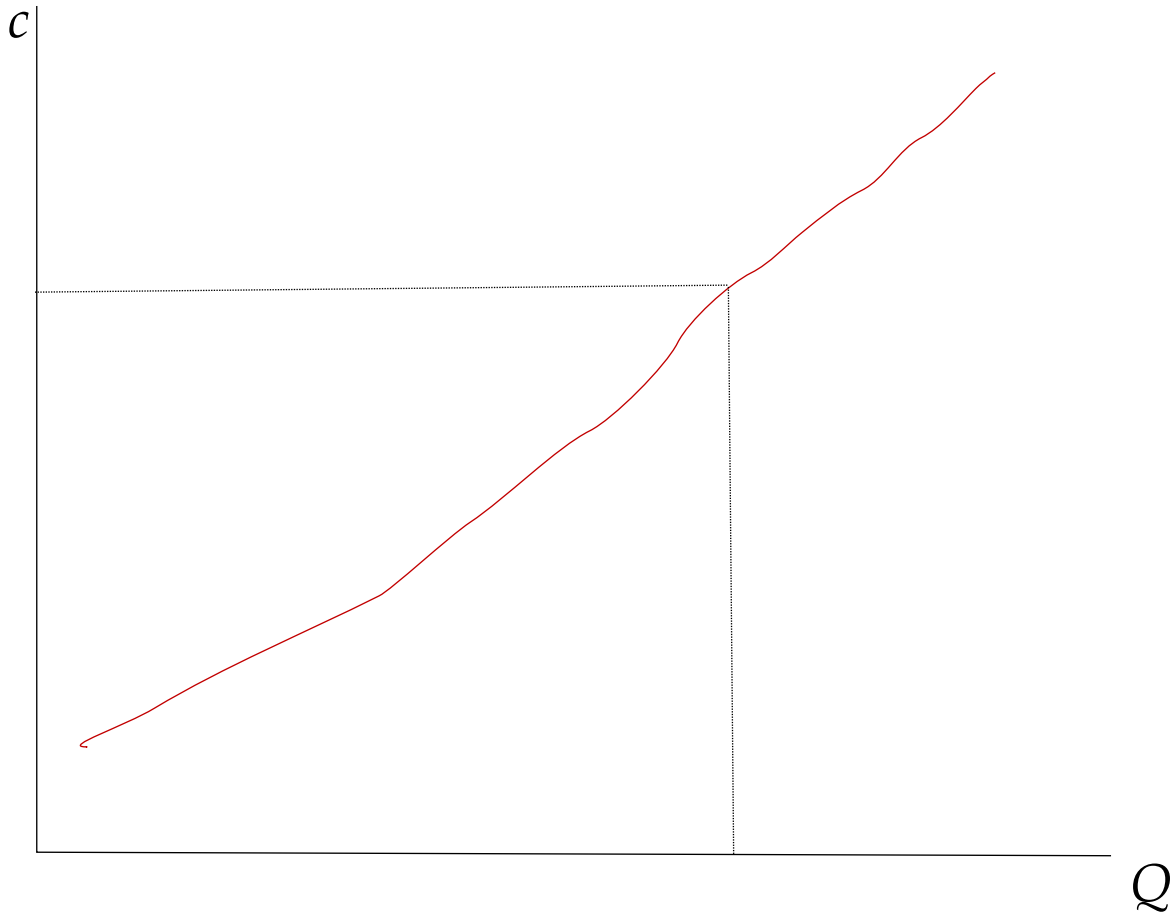
We will get a decreasing curve.

Very Large Markets



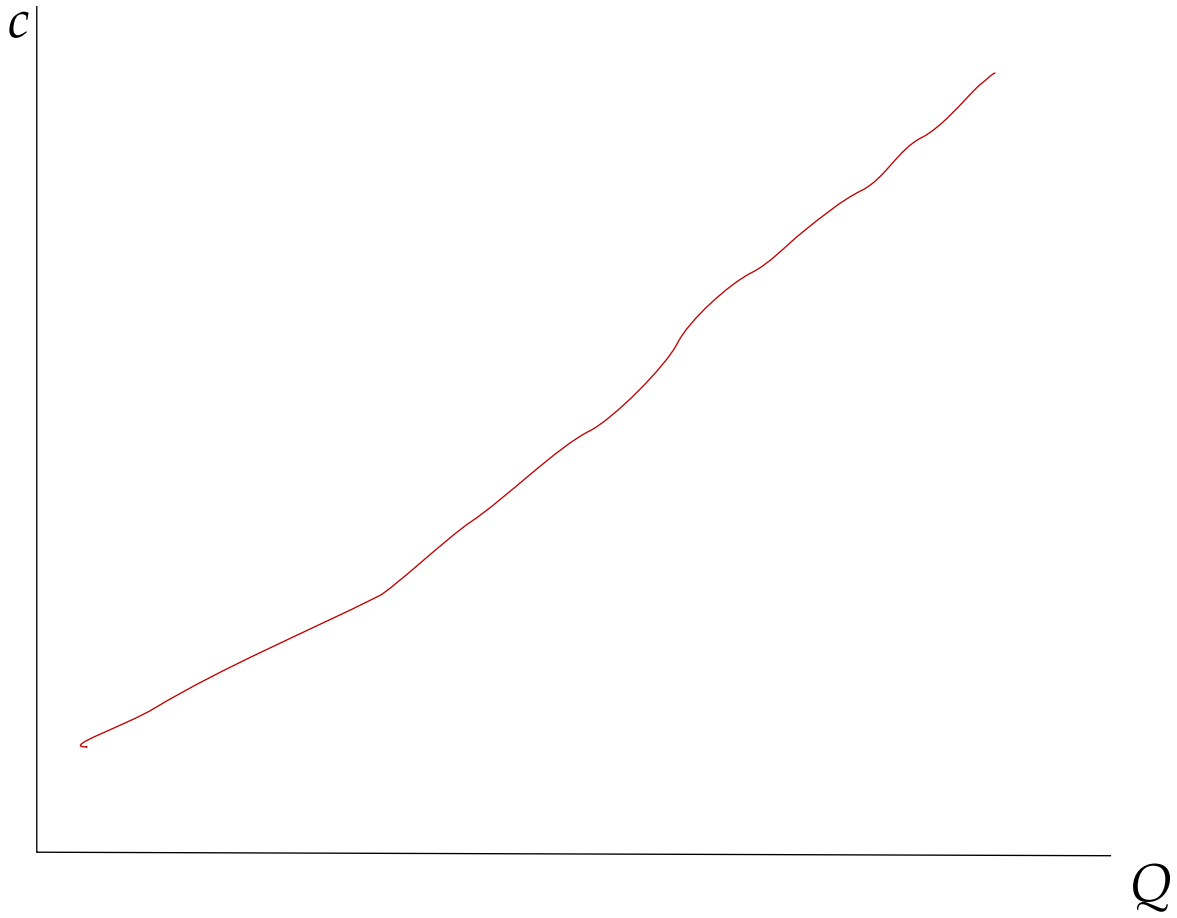
Suppose that there is a large number of sellers.

Very Large Markets



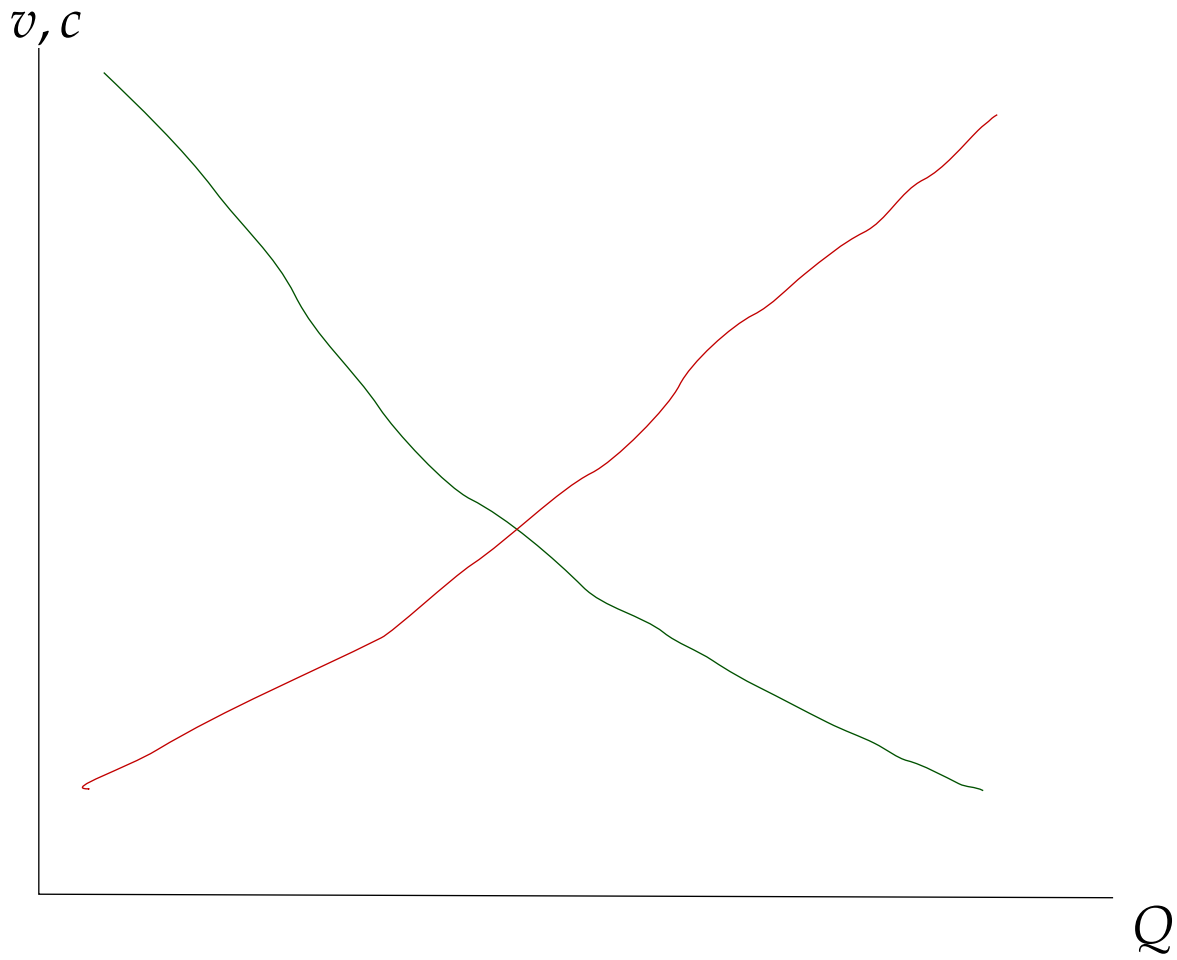
For any cost c, we can plot the number of buyers with costs less than c.

Very Large Markets



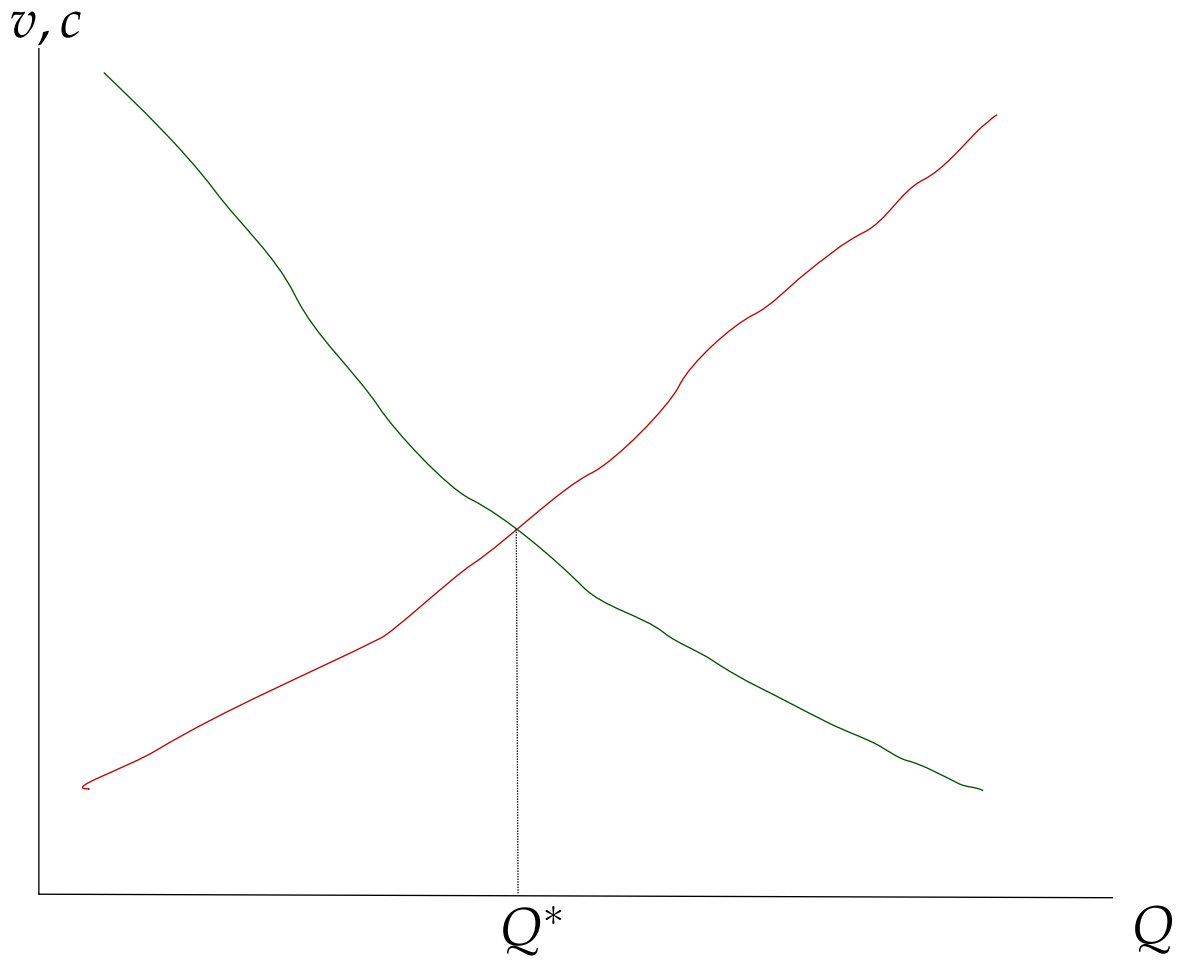
We will get an increasing curve.

Very Large Markets



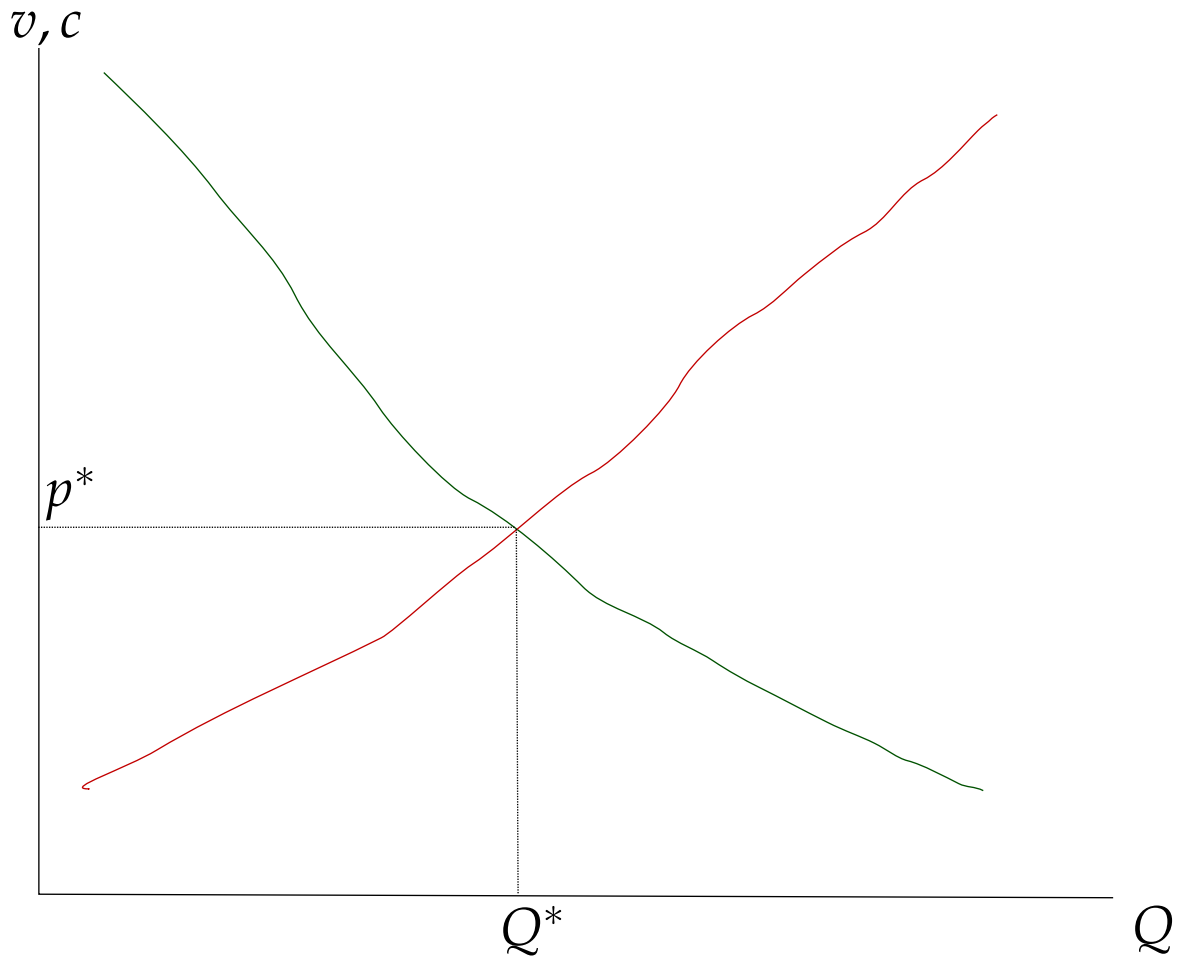
If we use the price discovery mechanism,

Very Large Markets



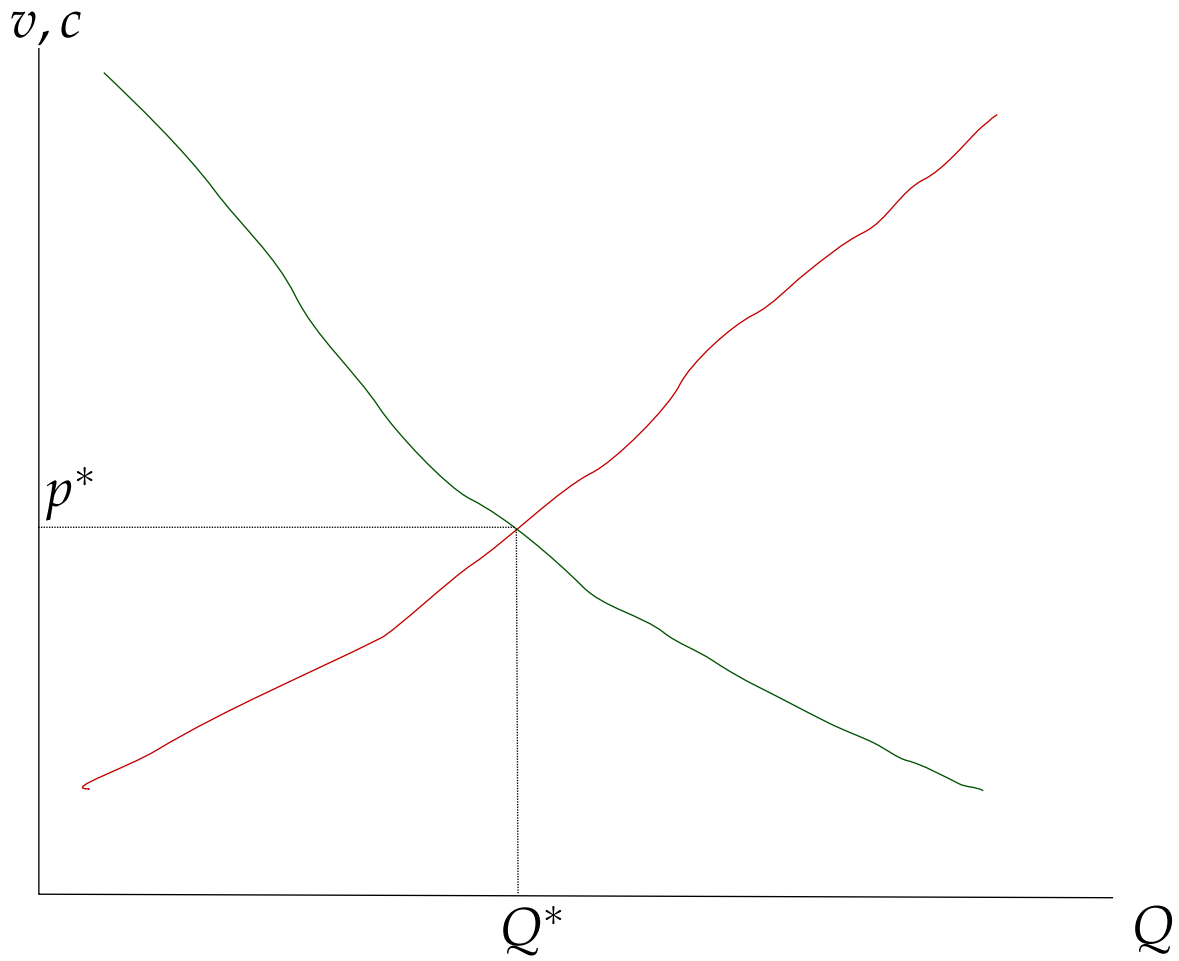
we will find that Q^* units should be traded.

Very Large Markets



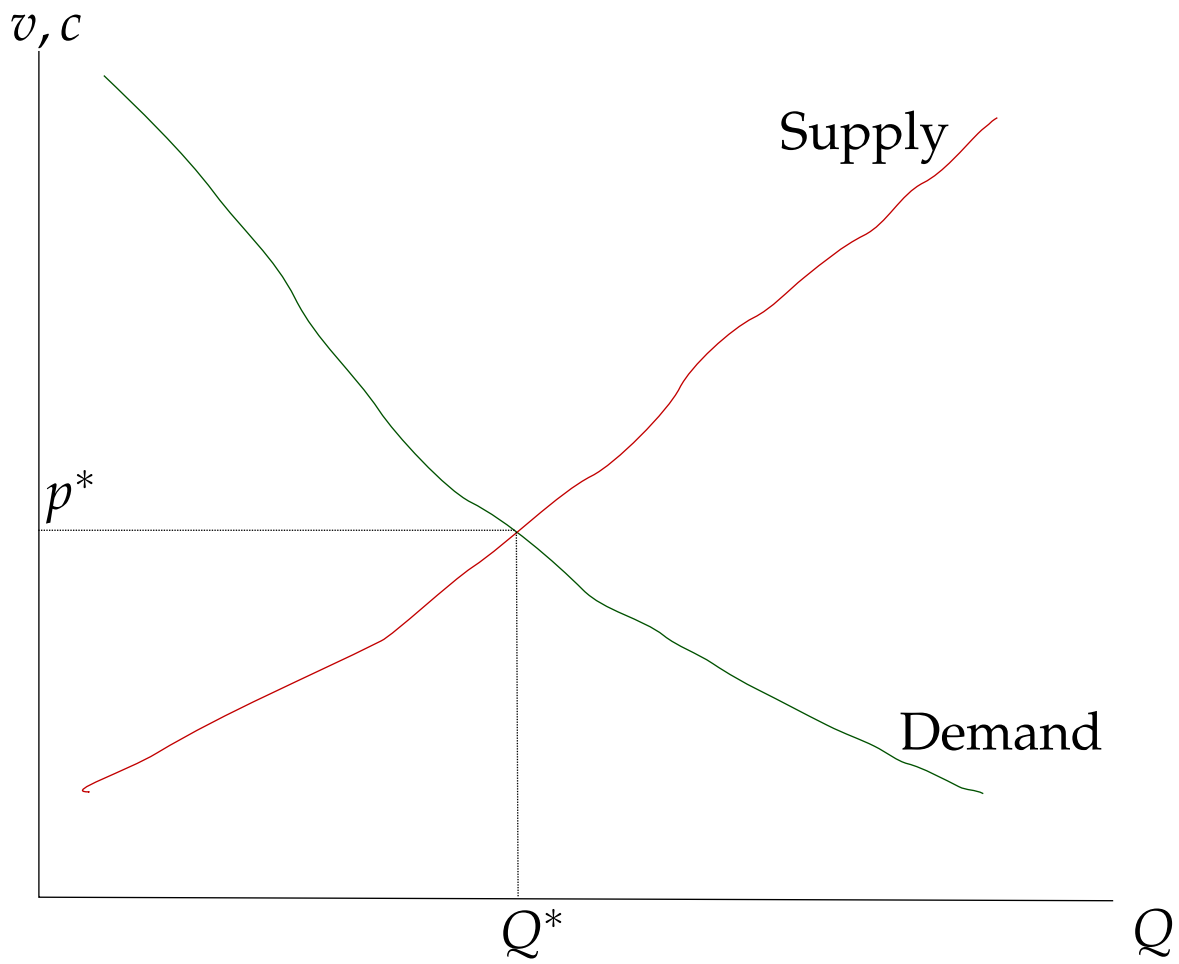
The sellers and buyers prices will virtually coincide at p^*.

Very Large Markets



This is the first-best.

Very Large Markets



And it clarifies why competitive markets are efficient.

Summary

- In small markets there is inefficiency in the form of too little trade.
- This is the cost of incentive-compatibility.
- In large markets, competition makes it easier and easier to solve this problem.
- In the limit, we achieve the first-best.
- However, this assumes there is some centralized institution which brokers trade between buyers and sellers.
- We are still left with the question of how trade works when the sellers design the mechanism rather than some benevolent planner.