Large Public Goods

Jeffrey Ely

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In small markets, trade is inefficient.
The same was true in the problem of providing public goods.
In large markets the inefficiency of trade disappeared.
We will investigate the efficiency of public goods in large “markets.”
Second-best mechanisms are rectangular.
Refresher: Fixed Cost-Sharing Mechanism

They ask each individual to contribute some fixed share of the cost. $\alpha_1 + \alpha_2 = c$. 
They are inefficient due to underprovision of the public good.
Each individual $i$ has a value $v_i$ for the public good.

As always, $v_i$ is known only to $i$.

Assume that the cost of the public good is $3/2$.

- So that $c = n/2$ where $n$ is the number of individuals.
- This will allow us to compare the inefficiency with the case of $n = 2$.
- We are assuming that the cost of providing the public good increases proportionally with the number of users.
Second-best Mechanism

- Just as in the case of 2 individuals, a second-best mechanism is fixed-cost sharing.
- “Cubic mechanism”
The set of possible values. Imagine a plane where $v_1 + v_2 + v_3 = c = 3/2$. It divides the cube in half.
A second-best mechanism is a 3-d rectangle above that plane.
The second-best mechanism is again a fixed cost-share mechanism. It is inefficient.

If $\alpha_1, \alpha_2, \alpha_3$ are the cost-shares then

- The size of the inefficiency can be measured by the ratio of
  - The volume of the region where the good is produced.
  - Over the volume of the region where the good should be produced.
- The numerator is $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3) \leq (1/2)^3$
- The denominator is 1/2.
- So the efficiency ratio is less than 1/4.
Efficiency Ratio with 2 individuals

- With two individuals and equal cost sharing the efficiency ratio is $1/2$.
- The production region has area $1/4$.
- The area of the region where the good should be produced is $1/2$ (same as with 3 individuals.)
- The inefficiency is \textit{worse} with three individuals than with two.
Efficiency Ratio with $n$ individuals

- With any number of individuals, a second-best mechanism will be a fixed cost-sharing mechanism.
- If the cost of the good is $n/2$, the region where the good should be produced will have volume $1/2$.
- The volume of the production region will be $\prod_{i=1}^{n}(1 - \alpha_i) \leq (1/2)^n$.
- The efficiency ratio is less than $(1/2)^{n-1}$.
- This gets small very fast and is close to zero for a large population.
Summary

- With public goods, the inefficiency gets worse as the size of the market grows.
- This is what makes public goods different from private goods.
- With private goods we can expect the competition from large markets to eliminate inefficiency.
- Not so with public goods.
- For this reason, public goods are usually provided by government, not decentralized markets.
- But some public goods are provided without government subsidies.
- This would make sense if the cost did not grow proportionally with the size of the population.
  - Public Radio
  - The arts.
- Or if the public good is excludable. (parks, beaches)