

Large Public Goods

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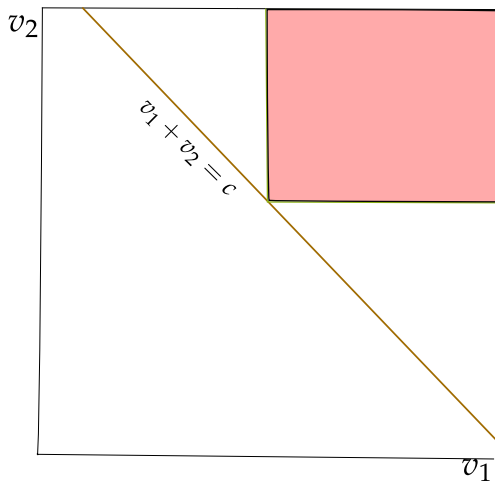
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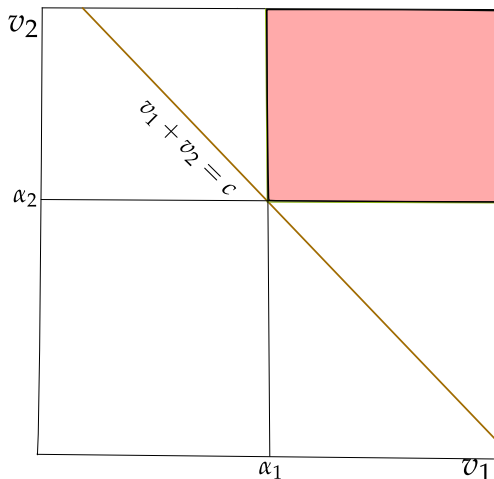
- In small markets, trade is inefficient.
- The same was true in the problem of providing public goods.
- In large markets the inefficiency of trade disappeared.
- We will investigate the efficiency of public goods in large “markets.”

Refresher: Fixed Cost-Sharing Mechanism



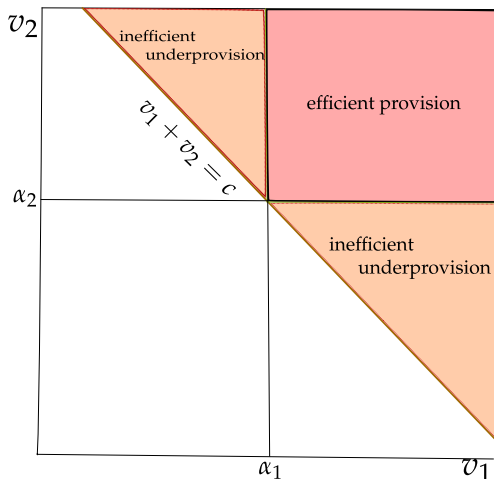
Second-best mechanisms are rectangular.

Refresher: Fixed Cost-Sharing Mechanism



They ask each individual to contribute some fixed share of the cost. $\alpha_1 + \alpha_2 = c$.

Refresher: Fixed Cost-Sharing Mechanism



They are inefficient due to *underprovision* of the public good.

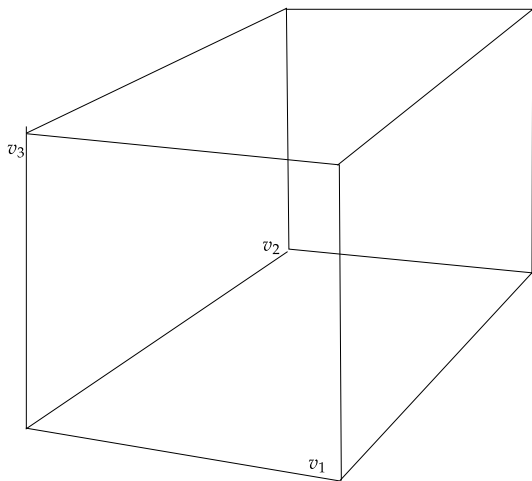
3 individuals

- Each individual i has a value v_i for the public good.
- As always, v_i is known only to i .
- Assume that the cost of the public good is $3/2$.
 - ▶ So that $c = n/2$ where n is the number of individuals.
 - ▶ This will allow us to compare the inefficiency with the case of $n = 2$.
 - ▶ We are assuming that the cost of providing the public good increases proportionally with the number of users.

Second-best Mechanism

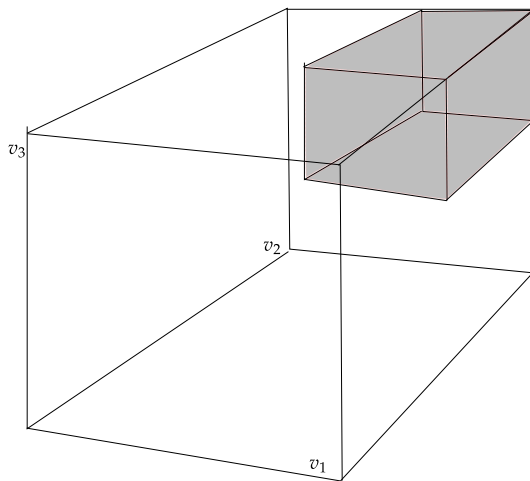
- Just as in the case of 2 individuals, a second-best mechanism is fixed-cost sharing.
- “Cubic mechanism”

Cubic Mechanism



The set of possible values. Imagine a plane where $v_1 + v_2 + v_3 = c = 3/2$. It divides the cube in half.

Cubic Mechanism



A second-best mechanism is a 3-d rectangle above that plane.

Inefficiency

- The second-best mechanism is again a fixed cost-share mechanism.
- It is inefficient.
- If $\alpha_1, \alpha_2, \alpha_3$ are the cost-shares then
 - ▶ The size of the inefficiency can be measured by the ratio of
 - ★ The volume of the region where the good is produced.
 - ★ Over the volume of the region where the good should be produced.
 - ▶ The numerator is $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3) \leq (1/2)^3$
 - ▶ The denominator is $1/2$.
 - ▶ So the efficiency ratio is less than $1/4$.

Efficiency Ratio with 2 individuals

- With two individuals and equal cost sharing the efficiency ratio is $1/2$.
- The production region has area $1/4$.
- The area of the region where the good should be produced is $1/2$ (same as with 3 individuals.)
- The inefficiency is *worse* with three individuals than with two.

Efficiency Ratio with n individuals

- With any number of individuals, a second-best mechanism will be a fixed cost-sharing mechanism.
- If the cost of the good is $n/2$, the region where the good should be produced will have volume $1/2$.
- The volume of the production region will be $\prod_{i=1}^n (1 - \alpha_i) \leq (1/2)^n$.
- The efficiency ratio is less than $(1/2)^{n-1}$.
- This gets small very fast and is close to zero for a large population.

Summary

- With public goods, the inefficiency gets *worse* as the size of the market grows.
- This is what makes public goods different from private goods.
- With private goods we can expect the competition from large markets to eliminate inefficiency.
- Not so with public goods.
- For this reason, public goods are usually provided by government, not decentralized markets.
- But some public goods are provided without government subsidies.
- This would make sense if the cost did not grow proportionally with the size of the population.
 - ▶ Public Radio
 - ▶ The arts.
- Or if the public good is *excludable*. (parks, beaches)