

# Profit Maximizing Mechanisms

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- To illustrate, we will analyze auctions as mechanisms for maximizing profits.
- And we will compare profit-maximizing auctions to efficient auctions.

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- Note that the seller has no direct reason to care about efficient allocation.

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  - ▶ The seller can control the reserve price.

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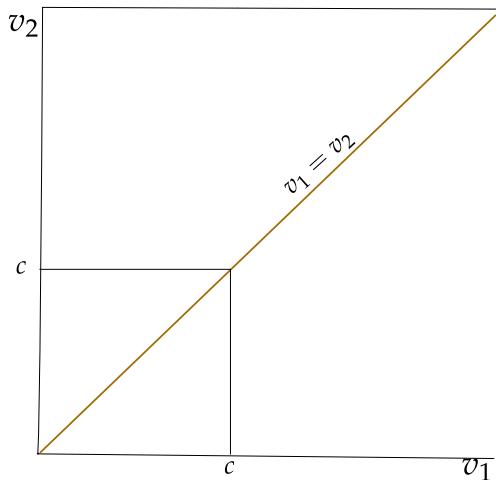
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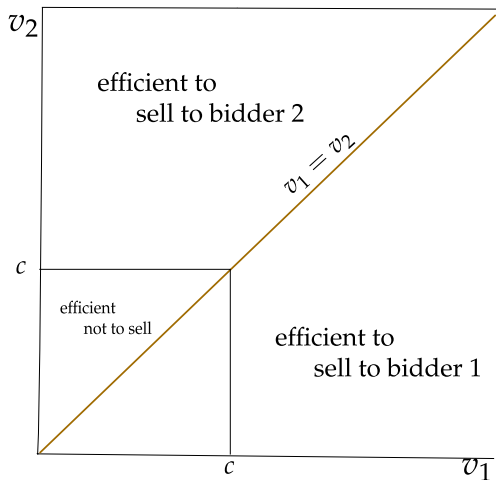
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  - ▶ The losing bidders pay nothing.

## Example: 2 Bidders



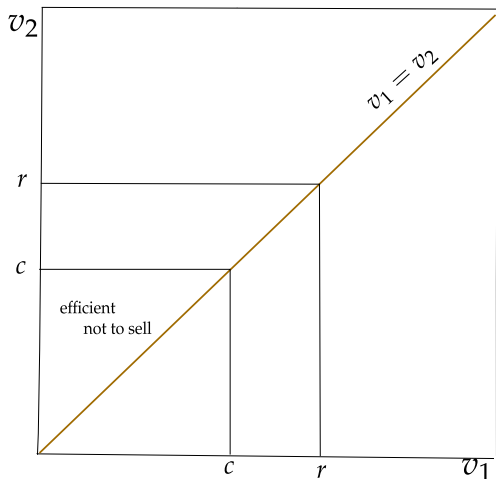
The bidders' values and the seller's cost.

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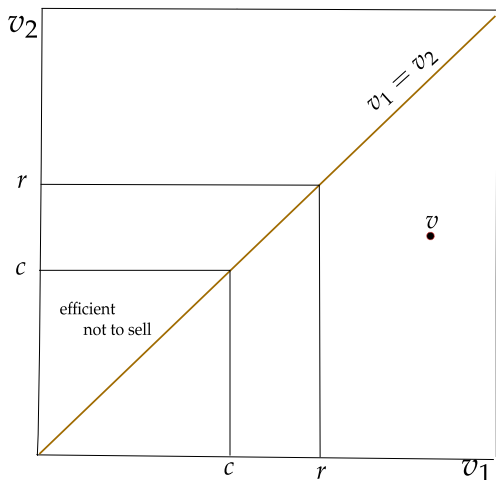
The utilitarian decision rule. This can be achieved by setting a reserve price  $r = c$ .

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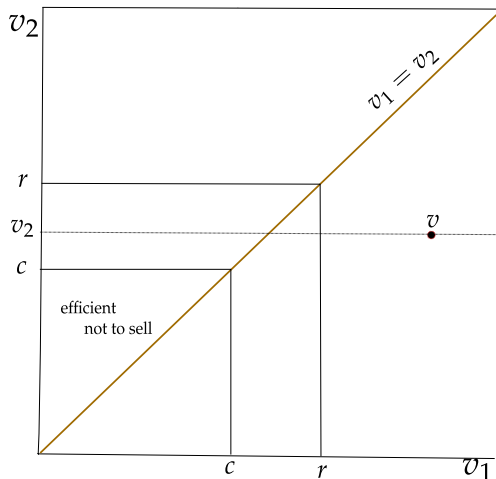
What if the seller used a reserve price higher than  $c$ ? (She would never use a lower reserve.)

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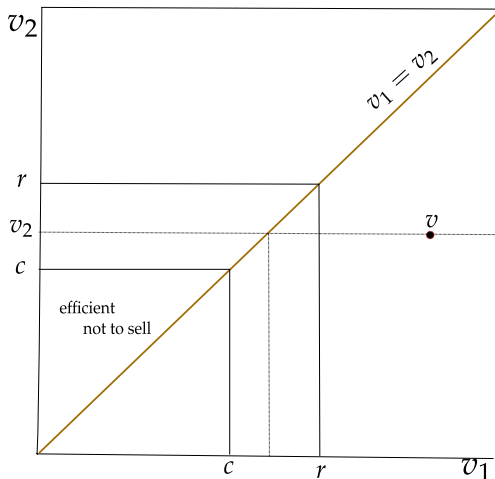
If the values are  $v = (v_1, v_2)$ , where  $v_1 > r$  but  $v_2 < r$  then bidder 1 wins.

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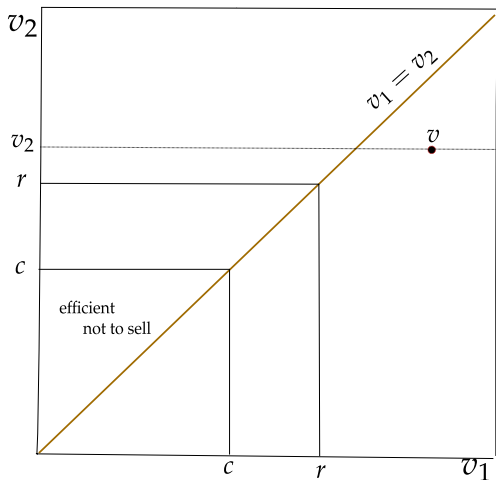
If the reserve price was  $c$ , then 1 would pay  $v_2$ .

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Since  $v_2 < r$ , with a reserve price of  $r$ , bidder 1 pays  $r$  instead. Good for the seller.

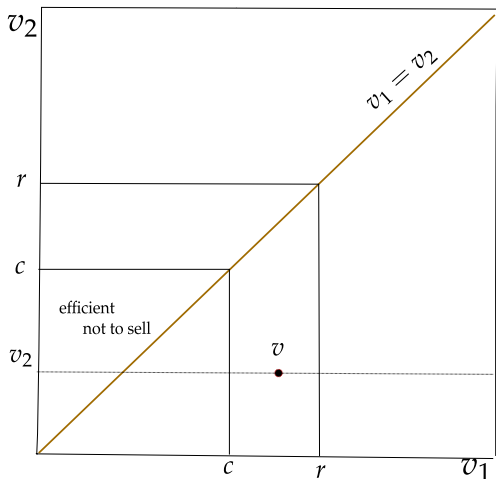
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If the losing bidder's bid is above  $r$ , then there is no difference between a reserve price of  $r$  vs  $c$ .

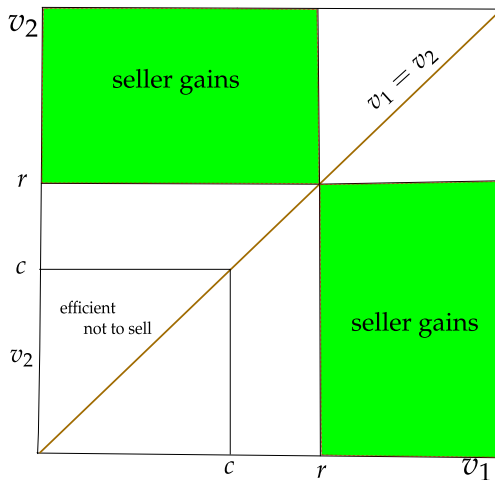


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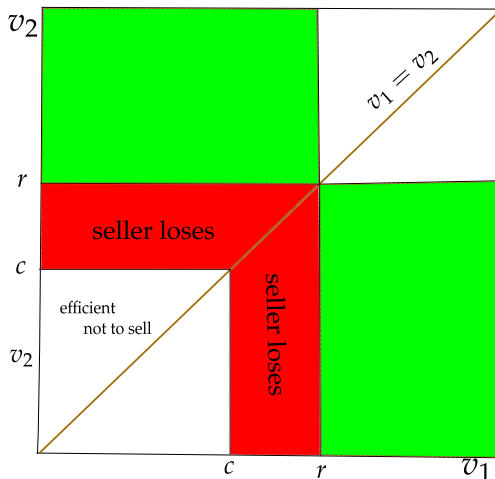
If the winning bidder's bid is less than  $r$  but greater than  $c$ , then the higher reserve price  $r$  winds up costing the seller a sale.

## Example: 2 Bidders



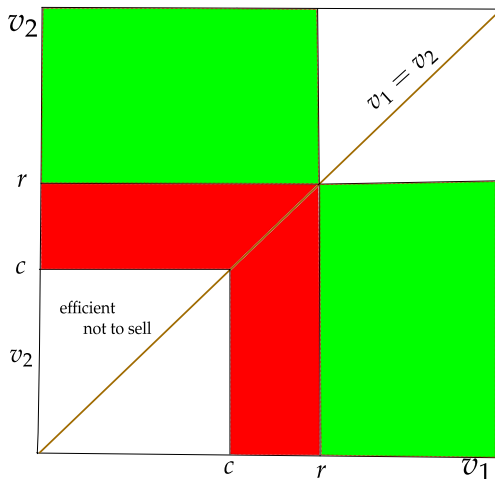
Here are all the cases where the seller increases profit by using the higher reserve price.

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Here are all the cases where the seller loses profit by using the higher reserve price.

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Notice that the buyers are *always* worse off from the higher reserve.

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- The seller's profit maximization problem is

$$\max_r \int_{v_1, v_2} \max\{0, \min\{v_1, v_2\} - r\} F(v) dv$$

FAIL



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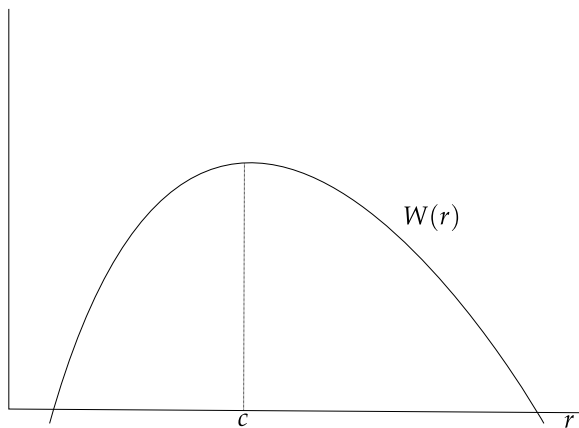
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- We know that
  - ▶ Total welfare is maximized by the utilitarian solution  $r = c$ .
  - ▶ The buyers' utility is unambiguously reduced by raising  $r$ .

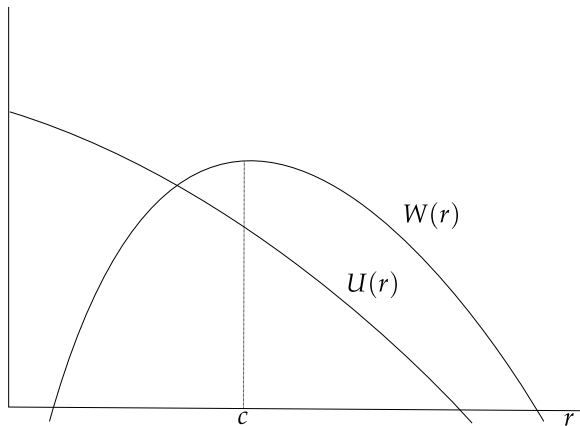


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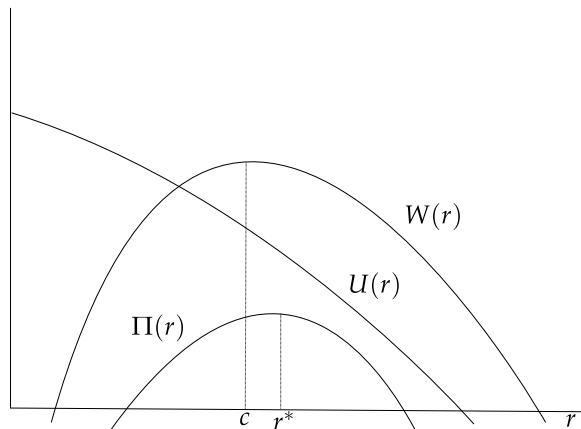
Total welfare is maximized at  $r = c$ . The curve is flat there.

$$r > c$$



Buyers' utility is decreasing.

$$r > c$$



This means that seller profit must be increasing at  $r = c$ .