Profit Maximizing Mechanisms

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To illustrate, we will analyze auctions as mechanisms for maximizing profits.

And we will compare profit-maximizing auctions to efficient auctions.
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- Each buyer $i$ has a value $v_i$ from obtaining the good.
- The seller does not know $v_i$. 

If the seller did know $v_i$ the problem would be simple.

Find the buyer with the highest $v_i$ and make a take-it-or-leave-it offer asking for price $v_i$.

We can think of the seller's problem as trying to design a mechanism so that

- The buyers have an incentive to tell the seller truthfully their values.
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  - The bidders have dominant strategies.
  - The seller can control the reserve price.
Reserve Prices

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- The losing bidders pay nothing.
Example: 2 Bidders

The bidders’ values and the seller’s cost.
Example: 2 Bidders

The utilitarian decision rule. This can be achieved by setting a reserve price \( r = c \).
What if the seller used a reserve price higher than $c$? (She would never use a lower reserve.)
Example: 2 Bidders

If the values are $v = (v_1, v_2)$, where $v_1 > r$ but $v_2 < r$ then bidder 1 wins.
Example: 2 Bidders

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Since $v_2 < r$, with a reserve price of $r$, bidder 1 pays $r$ instead. Good for the seller.
Example: 2 Bidders

If the losing bidder’s bid is above $r$, then there is no difference between a reserve price of $r$ vs $c$. 
Example: 2 Bidders

If the winning bidder’s bid is less than $r$ but greater than $c$, then the higher reserve price $r$ winds up costing the seller a sale.
Example: 2 Bidders

Here are all the cases where the seller increases profit by using the higher reserve price.
Example: 2 Bidders

Here are all the cases where the seller loses profit by using the higher reserve price.

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Notice that the buyers are *always* worse off from the higher reserve.
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The seller’s profit maximization problem is

$$\max_r \int_{v_1, v_2} \max\{0, \min\{v_1, v_2\} - r\} F(v) \, dv$$
A Simpler Approach

For our purposes we just want to know whether the seller will set $r = c$ or something higher.

- Define the total expected welfare (buyers’ utility plus seller’s profit) as $W(r)$.
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Total welfare is maximized at $r = c$. The curve is flat there.
Buyers’ utility is decreasing.
This means that seller profit must be increasing at $r = c$. 

\[ r > c \]