Swords or Plowshares? A Theory of the Security of Claims to Property

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February 28, 2012
Outline

1. Introduction/Motivation
2. The Model
3. Results
4. Extension
5. Conclusion
Objectives

Big objective: Develop a general equilibrium model for allocating between appropriative and productive activities.

- Differentiate between offensive and defensive resources
- Determine parameters for which there exists non-violent equilibria
- Welfare analysis based on security of claims to property
Findings

- Non-violence possible if war is sufficiently wasteful or if defenses are sufficiently effective
- Welfare analysis depends on initial wealth
  - Rich benefit from more secure property
  - Poor may benefit from easier stealing
- Paradox of power—in a world where violence is possible, poorer people are relatively better off
Consider a one-period model of production and appropriation

- Two agents, 1 and 2
- Agents have respective endowments $n_1$ and $n_2$
- Each agent $i$ can spend this endowment on three things:
  - Guns $g_i$, offensive weapons used for predation
  - Butter $k_i$, productive capital used to make consumable goods
  - Walls $h_i$, defensive weapons used against predation
Timing

- Time 1: Agents choose level of walls $h_i$
- Time 2: Agents choose gun level $g_i$ and butter level $k_i$
  - All variables subject to nonnegativity constraints
- Time 3: Production occurs; agent $i$ receives $\alpha k_i$, where $\alpha > 0$.
- Time 4: Any part of endowment subject to appropriation by the other agent
  - Agent $i$ retains $p_i$ of his endowment, where
    \[
    p_i = \frac{1}{1 + x_i}, \quad x_i = \frac{\theta g_j}{h_i},
    \]  
    where $\theta$ is an exogenous measure of offensive efficacy
  - Attacker $j$ acquires $(1 - \beta)(1 - p_i)$ of $i$’s endowment, where $\beta$ is an exogenous measure of the destructiveness of war
End result: agent $i$ has final nonnegative wealth

$$m_i = p_i n_i + (1 - \beta)(1 - p_j)n_j$$  \hspace{1cm} (2)

- Objective: maximize sum of consumables and final wealth, denoted $v_i$, so

$$v_i = \alpha k_i + m_i$$  \hspace{1cm} (3)
Let’s Solve This!

Backwards induction: Start with guns/butter decision

- NB: attack inevitable if $g_i > 0$

Fix levels of $h_i, h_j$ and note that $k_i = n_i - g_i - h_i$. Then take derivatives.

- We find that we either get an interior solution where

$$
\frac{\partial v_i}{\partial g_i} = -(1 - \beta) \frac{\partial p_j}{\partial g_i} n_j - \alpha = 0, \quad g_i > 0, \quad (4)
$$

or we get a corner solution where

$$
\frac{\partial v_i}{\partial g_i} = -(1 - \beta) \frac{\partial p_j}{\partial g_i} n_j - \alpha \leq 0, \quad g_i = 0. \quad (5)
$$
The Guns/Butter Decision

We can substitute for \( \frac{\partial p_j}{\partial g_i} \) to get

\[
g_i = \begin{cases} 
\sqrt{(1 - \beta) \frac{h_j n_j}{\theta \alpha}} - \frac{h_j}{\theta} & \text{for } 0 < h_j < h_j^* \\
0 & \text{for } h_j \geq h_j^*,
\end{cases}
\]

where

\[ h_j^* = (1 - \beta) \frac{\theta}{\alpha} n_j \]

We can look at \( h_j^* \) as the minimum investment in walls \( j \) has to make in order to deter \( i \) from attacking.
The Walls Decision

Now that we know how each player will act in Stage 2, we can look at Stage 1

- First, note from the previous guns/butter decision that as \( h_i \to 0 \),
  \[
  \frac{\partial v_i}{\partial h_i} \to \infty.
  \]
  - Spending nothing on walls allows \( j \) to steal everything with \( g_j = \epsilon \)

- Thus, we know that the \( h_i \geq 0 \) constraint isn’t binding

- Assuming that the \( h_i \leq n_i \) constraint isn’t binding either, we get an interior solution
  - Also means that marginal cost of walls is also \( \alpha \)
From before, we know that if \( h_j < h_j^* \), \( g_i \) is positive, but \( g_i = 0 \) otherwise.

If \( g_i = 0 \), then \( v_j \) is a decreasing linear function of \( h_j \).

Thus, \( v_j \) either has an interior maximum such that

\[
\frac{\partial v_j}{\partial h_j} = \left( \frac{\partial p_j}{\partial h_j} + \frac{\partial p_j}{\partial g_i} \frac{\partial g_i}{\partial h_j} \right) n_j - \alpha = 0, \quad \text{with} \ 0 < h_j < h_j^*,
\]

or we have a corner solution at \( h_j = h_j^* \) with

\[
\frac{\partial v_j}{\partial h_j} = \left( \frac{\partial p_j}{\partial h_j} + \frac{\partial p_j}{\partial g_i} \frac{\partial g_i}{\partial h_j} \right) n_j - \alpha > 0, \quad \text{for} \ h_j < h_j^*. \]
Substituting in expressions for $\frac{\partial p_j}{\partial h_j}$, $\frac{\partial p_j}{\partial g_i}$, and $\frac{\partial g_i}{\partial h_j}$, we get

$$h_j = \begin{cases} \frac{n_j}{4(1-\beta)\theta\alpha} < h_j^* & \text{for } 2(1-\beta)\theta > 1 \\ h_j^* = (1-\beta)\frac{\theta}{\alpha}n_j & \text{for } 2(1-\beta)\theta \leq 1 \end{cases}$$

(9)

But we have $h_j$ decreasing in $(1-\beta)\theta$ for $h_j < h_j^*$

- What’s going on here?
Plugging the previous equation into the guns decision, we get

\[ g_i = \begin{cases} \frac{1}{2\theta \alpha} \left(1 - \frac{1}{2(1-\beta)\theta}\right) n_j & \text{for } 2(1 - \beta)\theta > 1 \\ 0 & \text{for } 2(1 - \beta)\theta \leq 1 \end{cases} \]  

(10)

Plugging this into our contest success function, we get that

\[ p_j = p_i = p = \min \left[1, \frac{1}{2(1 - \beta)\theta} \right] \]  

(11)

Thus, we have fully secure claims to property \((p = 1)\) iff \(2(1 - \beta)\theta \leq 1\)
The Cost of Appropriative Activities

Using our results, we can look at the cost of appropriative activities (both offensive and defensive)

- Allocating resources to either guns or walls takes away from productive capital
- Total cost of appropriation is \( \alpha(h_i + h_j + g_i + g_j) \) plus any losses due to conflict
  - This component is \( \beta(1 - p)(n_i + n_j) \)
- Thus we can express the total cost of appropriation relative to the total endowment as a quantity \( c \), where

\[
c = \frac{\alpha(h_i + h_j + g_i + g_j) + \beta(1 - p)(n_i + n_j)}{n_i + n_j}
\]
The Cost of Appropriative Activities

Plugging in our previous results, we get

\[ c = \begin{cases} 
\beta + \frac{1}{\theta} \left(1 - \frac{1+\theta}{4(1-\beta)\theta}\right) & \text{for } 2(1 - \beta)\theta > 1 \\
(1 - \beta)\theta & \text{for } 2(1 - \beta)\theta \leq 1 
\end{cases} \quad (12) \]

Things to note:

- In nonaggressive equilibria, production losses come from allocating resources to walls
  - \( c \) is increasing in \( \theta \) and decreasing in \( \beta \)
- In equilibria with predation, these monotonic relations are lost
  - For \( p \) near 1, \( c \) is decreasing in \( p \)
  - For \( p \) far from 1, \( c \) approaches \( \beta \) as \( \theta \to \infty \)
Welfare Analysis

Similarly, we can get utility results in terms of the exogenous parameters by plugging in our $g_i$ and $h_i$ equations into our utility function and for $m_i$

$$v_i = \begin{cases} 
(\alpha + 1)n_i - \left(1 - \frac{1}{4(1-\beta)\theta}\right)n_i \\
\quad + (1 - \beta) \left(1 - \frac{1}{2(1-\beta)\theta}\right)n_j \text{ for } 2(1 - \beta)\theta > 1 \\
(\alpha + 1)n_i - (1 - \beta)\theta n_i \text{ for } 2(1 - \beta)\theta \leq 1
\end{cases}$$ (13)
For nonaggressive equilibria, we get that $v_i$ is decreasing in $\theta$ and increasing in $\beta$

For aggressive equilibria, things are more complicated

- No monotone relations to $\beta$ or $\theta$
- Depends on wealth of other player as well as one’s own
- For relatively rich players, ($n_i > n_j$), welfare always higher with lower $\theta$ and higher $\beta$
- But not so for relatively poor players!
  - With large $\theta$ and small $\beta$, a relatively poor agent’s utility is increasing in $\theta$ and decreasing in $\beta$
Relative Utilities

For nonaggressive equilibria, relative utilities $\frac{v_i}{n_i}$ and $\frac{v_j}{n_j}$ are equal

$$\frac{v_i}{n_i} = (\alpha + 1) - (1 - \beta)\theta = \frac{v_j}{n_j}$$

For aggressive equilibria, the poor get higher utility relative to their endowments. It’s increasing in the other person’s wealth.

- This gets us Hirshleifer’s paradox of power
Wrapup

- Can get either aggressive or nonaggressive equilibria depending on the efficacy of theft
- Cost/Welfare analysis straightforward for nonaggressive equilibria
  - Utility relative to wealth constant
  - Welfare increasing as theft becomes less attractive
- Analysis more complicated in aggressive models
  - Rich players like more secure claims to property
  - Poor players can appreciate the ability to steal effectively from their rich counterparts
Extending to Multiple Periods

Grossman and Kim (1996) extend this model to a multi-period dynastic model with one possible predatory agent and his prey

- Alterations/Restrictions:
  - Predator gets subscript \( a \), Prey gets subscript \( d \), everything gets a time superscript
  - Quantities \( g^t_d = h^t_a = 0 \)
  - Conflict outcome function now

\[
p^t = \begin{cases} 
1 - x^t & \text{for } 0 \leq x^t < 1 \\
0 & \text{for } x^t \geq 1,
\end{cases} \tag{14}
\]

where \( x^t = \theta \frac{g^t_a}{h^t_d} \)

- Predator observes all of Prey’s actions before making decisions
Agents allocate between productive capital, allocative capital, and consumption (denoted $c_i^t$)

- Agents care about their consumption and their bequest to next generation. Thus agent $i$’s utility is given by

$$v_i^t = u_i(c_i^t) + z_i(n_{i}^{t+1}),$$  \hspace{1cm} (15)

where both $u$ and $z$ are increasing and concave

- If there is thievery on the part of Predator, then that gets factored into the next generation’s endowment

$$n_{a}^{t+1} = \alpha k_{a}^t + (1 - \beta)(1 - p^t)\alpha k_{d}^t$$  \hspace{1cm} (16)

- Prey’s bequest is simply what Predator doesn’t steal

$$n_{d}^{t+1} = p^t \alpha k_{d}^t$$  \hspace{1cm} (17)
Predator Behavior

Predator can either invest in productive capital to build his bequest or steal capital from Prey

- Both Prey’s defenses and his capital are known to Predator
- Effects of both investment and theft on bequest are constant

**Investment:** $\alpha$

**Theft:** $(1 - \beta) \frac{\theta}{h_d^t} \alpha k_d^t$

- We can determine our threshold level of $h_d^t$ as $(1 - \beta) \theta k_d^t$
Predator Behavior

Predator will take an all-or-nothing approach to this business

- If $h_d^t < (1 - \beta)\theta k_d^t$, then
  
  $$ k_a^t = 0, \quad g_a^t = n_a^t - c_a^t, $$

  and $c_a^t$ is chosen such that

  $$ \frac{\partial v_a^t}{\partial c_a^t} = u'_a(c_a^t) - (1 - \beta)\frac{\theta}{h_d^t} \alpha k_d^t z'_a(n_{a+1}^t) = 0 $$

  (19)

- If $h_d^t \geq (1 - \beta)\theta k_d^t$, then

  $$ k_a^t = n_a^t - c_a^t, \quad g_a^t = 0, $$

  and $c_a^t$ is chosen such that

  $$ \frac{\partial v_a^t}{\partial c_a^t} = u'_a(c_a^t) - \alpha z'_a(n_{a+1}^t) = 0 $$

  (21)
We can determine that Prey’s utility has one or two local maxima

- Utility is always locally maximized where Predator is barely deterred, that is

\[ h_d^t = (1 - \beta) \theta k_d^t \]  
\[ \frac{\partial v_d^t}{\partial c_d^t} = u'_d(c_d^t) - \frac{\alpha}{1 + (1 - \beta) \theta} z'_d(n_d^{t+1}) = 0 \]  
\[ k_d^t = n_d^t - c_d^t - h_d^t \]
Prey Behavior

Prey may also elect to allow some predation. This requires a local maximum such that

\[
\frac{\partial v_d^t}{\partial h_d^t} = \left(-p^t \alpha + \frac{\partial p^t}{\partial h_d^t} \alpha k_d^t \right) z_d'(p^t \alpha k_d^t) = 0, \tag{25}
\]

and

\[
\theta c_a^t < h_d^t < (1 - \beta) \theta k_d^t
\]

\[
\frac{\partial v_d^t}{\partial c_d^t} = u_d'(c_d^t) - p^t \alpha z_d'(p^t \alpha k_d^t) = 0 \tag{26}
\]

\[
k_d^t = n_d^t - c_d^t - h_d^t, \tag{27}
\]

where \( c_a^t \) is the optimal level of consumption determined by Predator.
Basic Dynamics

Prey may tolerate some predation for a very poor Predator

- Limited resources means that Predator can only steal so much

As Predator gets richer, however, Prey would lose too much from appeasement and goes on to deterrence

- Limited window for which allowing predation is feasible
  - Size of this window is decreasing in $c_a^t$
  - Predator’s consumption is increasing in his endowment for most utility functions

- Thus, the toleration/deterrence decision is determined by the relative wealth levels of each player
Toleration/Deterrence

Predator dynasty grows weakly faster than prey dynasty

- If $\theta$ is zero, then neither agent allocates resources to weaponry, and each grows at the same constant rate $r$ (under log utility, $\frac{\alpha}{2}$).
- With positive $\theta$, Prey decides to allocate some resources to defense, lowering the rate of capital growth.
  - If Prey decides to tolerate theft, then Predator accumulates capital at a rate greater than $r$.
  - If Prey decides to deter theft, then Predator accumulates capital at rate $r$.
- If someone in the Prey dynasty tolerates theft, then a later member will deter it.
Predator has two forms of capital accumulation technology—investment and theft

- Prey may tolerate theft if complete deterrence is excessive
- Theft leads to wealth gap closing, and theft becomes more costly for Prey to allow
- Result: eventual deterrence of theft
Take-Away Points

One-period model:
- Peace is possible if efficacy of attacking is low enough and destruction from war is high enough
- Peaceful equilibria give more utility to richer people
- In aggressive equilibria, poorer agents may enjoy less secure claims to property

Dynamic model:
- Theft allowable if predatory agent is weak enough
- Predator gets too strong to tolerate eventually; deterrence by the defensive agent is inevitable
• Presents realistic addition to standard guns-and-butter framework
• Calculations yield intuitive results
• Additional extension: more than two agents
  • Possibility of subsets of agents being nonaggressive towards one another
  • Main issue: finding characteristics of nonaggressive subsets of agents